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On exponential stability analysis for neural networks with time-varying delays and general activation functions

Yijing Wang [a], Cuili Yang [b], Zhiqiang Zuo [a,*]

[a] Tianjin Key Laboratory of Process Measurement and Control, School of Electrical Engineering and Automation, Tianjin University, Tianjin 300072, PR China
[b] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong

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ABSTRACT

This paper is concerned with the exponential stability analysis for a class of cellular neural networks with both interval time-varying delays and general activation functions. The boundedness assumption of the activation function is not required. The limitation on the derivative of time delay being less than one is relaxed and the lower bound of time-varying delay is not restricted to be zero. A new Lyapunov–Krasovskii functional involving more information on the state variables is established to derive a novel exponential stability criterion. The obtained condition shows potential advantages over the existing ones since no useful item is ignored throughout the estimate of upper bound of the derivative of Lyapunov functional. Finally, three numerical examples are included to illustrate the proposed design procedures and applications.

1. Introduction

In the past few years, cellular neural networks (CNNs) have been well studied owing to their wide applications in optimization, image processing, fixed point computations and so on [1–3]. The applications of neural networks depend on their dynamical behavior. Therefore, the dynamical analysis of the networks is necessary for practical design of neural networks. On the other hand, in the electronic implementation of analog neural networks, time delays occur in the communication and response of neurons due to the finite switching speed of amplifier. It is known that time delay can influence the stability of a network by creating oscillatory or unstable phenomena. Therefore, the study of neural networks with consideration of time delays has received considerable attention for a long time (see e.g., [4–19] and the references therein).

As we know, exponential stability is a more favorite property than asymptotic stability since it gives a faster convergence rate to the equilibrium point. In [30], Liao et al. pointed out that the property of exponential stability is particularly important when the exponential convergence rate is used to determine the speed of neural computations. Consequently, great efforts have been made to exponential stability analysis for neural networks with constant or time-varying delays. For example, delay-dependent criteria were derived by making use of information on the length of delay in [20–27]. In [28–32], neural networks with constant delays were considered and some sufficient conditions on the global exponential stability were obtained.

On the other hand, time varying delays in neuron signals are often inevitable in many engineering applications and hardware implementations of neural networks because of the finite switching speed of amplifiers in electronic neural networks, or the finite signal propagation time in biological networks. In addition, it is important to achieve global exponential stability to increase the convergence rate of neural networks tending toward equilibria. Therefore, the global exponential stability of

* Corresponding author.
E-mail address: zqzuo@tju.edu.cn (Z. Zuo).

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neural networks with time-varying delays deserves in-depth investigation. What we concern is how to derive better exponential stability criteria by choosing appropriate Lyapunov–Krasovskii functional and avoiding any procedure which may bring conservatism when calculating the upper bound of time derivative of Lyapunov–Krasovskii functional. In [30], the global exponential stability of a general class of neural networks with time-varying delays was addressed by the approach combining the Lyapunov–Krasovskii functionals with the linear matrix inequality. Wu et al. [21] derived an exponential stability criterion by considering the relationship between the time-varying delay, its lower and upper bounds. The robust exponential stability of neural networks with multiple delays was studied in [31]. Their results are then generalized to the interval neural networks and bidirectional associative memory (BAM) neural networks. By applying the idea of vector Lyapunov function, Zheng et al. [23] proposed some sufficient conditions, which generalizes some previous criteria. But some negative terms in the derivative of the Lyapunov functional tend to be ignored, and this may bring conservativeness to some extent.

Motivated by the above discussions, the objective of this paper is to further investigate the exponential stability of the CNNs with interval time-varying delays and general activation functions. Here we remove the boundedness assumption of the activation function, relax the limitation on the derivative of time delay being less than one and the lower bound of time-varying delay being zero. Inspired by the delay center point method in [33], here we construct a more general Lyapunov–Krasovskii functional by utilizing the central point of the lower and upper bounds of delay. Since more information is involved and no useful item is ignored throughout the estimate of upper bound of the derivative of Lyapunov functional, the developed conditions are expected to be less conservative than the previous ones. Note that all the conditions are expressed in terms of LMIs, which can be efficiently solved by the interior point method [34].

The rest of this paper is organized as follows. Section 2 formulates the problem and gives some preliminaries. The main results are derived in Section 3. In Section 4, three examples are provided to demonstrate the effectiveness of this method. Finally, some concluding remarks are drawn in Section 5.

2. Problem formulation and preliminaries

Let us consider the following continuous-time CNNs with interval time-varying delays:

$$\dot{u}(t) = -Au(t) + Bg(u(t)) + Wg(u(t-d(t))) + J \tag{1}$$

where $u(t) = [u_1(t)\ u_2(t)\ \dots\ u_n(t)]^T \in R^n$ is the neural state vector. $A = diag\{a_1, a_2, \dots, a_n\} > 0$ is the state feedback coefficient matrix; $g(u(t)) = [g_1(u_1(t)), g_2(u_2(t)), \dots, g_n(u_n(t))]^T$ is the activation of neurons. $B = (b_{ij})_{n\times n}$ and $W = (w_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $J = [J_1\ J_2\ \dots\ J_n]^T$ represents the external inputs and $d(t)$ is the time-varying delay.

Assumption 1. The delay $d(t)$ is time-varying and satisfies:

$$d_1 \leqslant d(t) \leqslant d_2; \quad \dot{d}(t) \leqslant \mu$$

where $0 \leqslant d_1 < d_2$ and $\mu > 0$ are known constants.

Assumption 2. The activation function $g_i(\cdot)$, $(i = 1, 2, \dots, n)$ satisfies

$$\sigma_i^- \leqslant \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leqslant \sigma_i^+ \tag{2}$$

for any $s_1, s_2 \in R$, $s_1 \neq s_2$, where σ_i^- and σ_i^+ are known constants.

Moreover, we assume that the initial condition of system (1) has the form

$$u(t) = \phi(t), \quad t \in [-d_2, 0]$$

where function $\phi(t)$ is continuous.

Then, by using the well-known Brouwers fixed-point theorem, one can easily prove that there exists at least one equilibrium point for system (1). For the sake of simplicity in the exponential stability analysis of system (1), we make the transformation $x(\cdot) = u(\cdot) - u^*$, then we have

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Wf(x(t-d(t))) \tag{3}$$

where $x(t) = [x_1(t), \dots, x_n(t)] \in R^n$ is the state vector of the transformed system and u^* is an equilibrium point of system (1). Note that $f_j(x_j(t)) = g_j(x_j(t) + u_j^*) - g_j(u_j^*)$ with $f_j(0) = 0$, $(j = 1, 2, \dots, n)$. From condition (2), $f_j(\cdot)$ satisfies the following condition:

$$\sigma_j^- \leqslant \frac{f_j(s)}{s} \leqslant \sigma_j^+ \quad \forall s \in R, \quad f_j(0) = 0, \quad (j = 1, 2, \dots, n). \tag{4}$$

It is obvious that the equilibrium point of system (1) is exponential stable if and only if the zero solution of system (3) is exponential stable.