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# Numerical simulation of a flexible fiber deformation in a viscous flow by the immersed boundary-lattice Boltzmann method

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ABSTRACT

In this paper, deformation of a mass-less elastic fiber with a fixed end, immersed in a two-dimensional viscous channel flow, is simulated numerically. The lattice-Boltzmann method (LBM) is used to solve the Newtonian flow field and the immersed-boundary method (IBM) is employed to simulate the deformation of the flexible fiber interacting with the flow. The results of this unsteady simulation including fiber deformation, fluid velocity field, and variations of the fiber length are depicted in different time-steps through the simulation time. Similar trends are observed in plots representing length change of fibers with different values of stretching constant. Also, the numerical solution reaches a steady state equivalent to the fluid channel flow over a flat plate.



## 1. Introduction

A large number of problems in fluid and biofluid dynamics involve interactions between deformable elastic bodies and incompressible viscous fluids [1]. Blood flow over heart valve leaflets, red blood cells motions and deformations in arterioles and capillaries, vibrations of the basilar membrane of the inner ear, and fly of insects are some examples of these fluid–structure interaction (FSI) problems. The immersed boundary method has been realized as an effective scheme for the simulation of different problems that involve such interactions between fluids and elastic structures. This method was first introduced to model and simulate the blood flow in human heart and dynamics of heart valves [2–6]. In this method, the fluid equations are discretized on a fixed Eulerian grid over the entire domain and the immersed membrane is discretized on a moving Lagrangian array of points that contribute to the force term in the fluid equations. The fluid-membrane interaction is carried out via a Dirac delta function [7]. IBM is very powerful in terms of computational efficiency, regardless to the geometry of the immersed structure. This method has been applied to various problems in computational biofluid mechanics such as simulation of red blood cells deformation [8], aggregation of platelets during blood clotting [9–11], swimming of eels, bacteria and sperm [12–14], waving motions of cilia [15], flow in elastic blood vessels [16] and fly of insects [17].

The lattice-Boltzmann method is originated from the lattice gas (LG) automata, a discrete particle kinetics method emanating from Boltzmann's kinetic theory of gases. In this method, time and space are discretized to solve the lattice-Boltzmann equation for particle velocity distribution functions. In the last decades, this method has developed successfully as a new and powerful alternative approach for solving the Navier–Stokes equations [18]. Its remarkable capability in simulation of complex fluid physics has immensely raised researchers' interest in the method in the past years.

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There has been an increasing interest in combining LBM and IBM to solve FSI problems particularly in the field of biofluid mechanics. Feng and Michaelides [19] introduced an IB–LB scheme to simulate the sedimentation of a large number of circular particles in an enclosure containing a viscous fluid. Zhang et al. [20,21] have employed this method to model the deformable liquid capsules suspended in a viscous fluid flow to simulate the blood flow in microcirculation. Simulation of red blood cells aggregation and dissociation in shear flows using IB–LBM has been performed by these scholars in another publication [22]. Cheng and Zhang [23] proposed an IB–LB scheme, suitable for rapid boundary motion and large pressure gradient FSI to simulate the diastolic filling jet flow between mitral leaflets in a simplified 2D left heart model.

Furthermore, Zhang et al. [24] performed an experimental study on the dynamics of flexible filaments in a flowing soap film as a two-dimensional version of the flag-in-wind problem. Zhu and Peskin [25] numerically simulated the same problem by the immersed boundary method. Their model consisted of a one-dimensional immersed moving boundary with one tethered end in a two-dimensional laminar flow. The boundary exerts elastic forces to the film and moves at the local film velocity. In addition, multigrid method was used to solve the discretized fluid equations, due to the non-uniform density throughout the solution domain. Their results, which were in accordance with the experimental data obtained by Zhang et al. [24], indicated that the sustained flapping of the filament only occurs when its mass is included in the formulation of the model. Within a certain range of mass, the more the mass of the filament, the bigger the amplitude of the flapping. They, also, showed that when the length of filament is short enough (below a critical length), the filament always approaches its straight (rest) state in which it points downstream; however, when the length is larger, the system is bistable, which means it can settle into either state (rest state or sustained flapping) depending on the initial conditions. Zeng et al. [26] employed a mixed Euler–Lagrange approach together with a bead-elastic rod model to simulate motions of a fiber in airflow. Their proposed model is applicable to textile process in which the interactions of a fiber and airflow in the nozzle of an air-jet spinning machine are of importance. Zhu [27] simulated the interactions of an elastic fiber anchored at its centre point and immersed in a flowing viscous incompressible fluid by IBM to investigate the influence of Reynolds number, fiber flexure modulus, and fiber length on vortex shedding. Moreover, Zhu and Peskin [28] numerically studied the drag of a flexible fiber in a 2D viscous incompressible flow using IBM. In this study, the shape of the fiber and the drag were computed as a function of inflow speed, fiber length, bending rigidity, and fiber mass density. Hao and Zhu [29] proposed an LB based implicit IBM to obviate the drawbacks of IB–LB explicit schemes in terms of stability and restriction of time step. The test case in their study was a flexible filament in a flowing viscous fluid.

In this paper, the LBM is used to simulate the viscous flow of a Newtonian incompressible fluid in a rectangular channel over a flexible fiber immersed in the flow field. The fiber has a tethered end and is initially placed at a fixed angle. The IBM is used to model the deformation of the elastic fiber and its interactions with the fluid. Such a system (a flexible fiber with a fixed end in a viscous flow) is a two-dimensional version of the flag-in-wind problem if the effects of fiber inertial forces due to its mass are taken into account. With some modifications and developments, this model can also be used to study swimming motions of creatures such as eels and sperms.

## 2. Mathematical formulation and numerical model

The numerical scheme employed in this paper is according to the explicit IB–LBM explained in Hao and Zhu [29]. Consider an elastic structure in the flow of an incompressible viscous fluid. The dimensionless general form of the Boltzmann equation with BGK approximation for the fluid–solid interaction is as follows:

$$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t}+\xi\cdot\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{x}}+\mathbf{f}_{ib}(\mathbf{x},t)\cdot\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{v}}=-\frac{1}{\tau}(f(\mathbf{x},\mathbf{v},t)-f^{(0)}(\mathbf{x},\mathbf{v},t)). \quad (1)$$

Eq. (1) is the BGK equation which is used to describe the motion of the fluid and the immersed boundary. $f(\mathbf{x},\mathbf{v},t)$ is the single particle distribution function, where $\mathbf{x}$ is the spatial coordinate, $\mathbf{v}$ is particle velocity, and $t$ is the time. $f(\mathbf{x},\mathbf{v},t)d\mathbf{x}d\mathbf{v}$ represents the possibility of finding a particle at time $t$ in the location of $[\mathbf{x},\mathbf{x}+d\mathbf{x}]$ with a velocity between $\mathbf{v}$ and $\mathbf{v}+d\mathbf{v}$. The term $1/\tau(f-f^{(0)})$ is known as the BGK approximation for the complex collision operator in the Boltzmann equation, where $\tau$ is the dimension-less relaxation time. This parameter is related to the dimensionless kinematic viscosity in LBM. $f^{(0)}$and $\mathbf{f}_{ib}(\mathbf{x},t)$ are the Maxwellian distribution and the external force term that is exerted to the fluid by the immersed boundary, respectively. One can compute the macroscopic variables such as mass density ($\rho$) and momentum density ($\rho\mathbf{u}$) from the velocity distribution function via Eqs. (2) and (3).

$$\rho(\mathbf{x},t)=\int f(\mathbf{x},\mathbf{v},t)d\mathbf{v}, \quad (2)$$

$$(\rho\mathbf{u})(\mathbf{x},t)=\int f(\mathbf{x},\mathbf{v},t)\mathbf{v}d\mathbf{v}. \quad (3)$$

The Eulerian force density $\mathbf{f}_{ib}(\mathbf{x},t)$ defined on the Eulerian grid is computed from the Lagrangian force density $\mathbf{F}(\alpha,t)$ defined on the Lagrangian grid by Eq. (4).

$$\mathbf{f}_{ib}(\mathbf{x},t)=\int \mathbf{F}(\alpha,t)\delta(\mathbf{x}-\mathbf{X}(\alpha,t))d\alpha, \quad (4)$$

where $\alpha$ is a measure of the arc length along the fiber, and $\delta(\mathbf{x})$ is the Dirac delta function. The Lagrangian force density $\mathbf{F}$ is computed as follows: