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Letter to the Editor

Comment on "Based on interval type-2 adaptive fuzzy H^{∞} tracking controller for SISO time-delay nonlinear systems"

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ABSTRACT

In this comment, we point out the inappropriateness of Theorem 1 in the article [Tsung-Chih Lin, Mehdi Roopaei. Based on interval type-2 adaptive fuzzy H^{∞} tracking controller for SISO time-delay nonlinear systems. Commun Nonlinear Sci Numer Simulat 2010;15:4065–75]. For solving this problem, some formular mistakes are corrected and novel parameter adaptive laws of interval type-2 fuzzy neural network system are given.

1. Introduction

In [1], consider the nth-order time-delay system as follows:

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = f(\boldsymbol{x}, \boldsymbol{x}(t-\tau_1), \ldots, \boldsymbol{x}(t-\tau_r)) + g(\boldsymbol{x}, \boldsymbol{x}(t-\tau_1), \ldots, \boldsymbol{x}(t-\tau_r))u(t) + d(t), \\ y = x_1, \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where f and g are unknown functions, u and y are control input and system output, respectively, $\boldsymbol{x} = [x_1, \ldots, x_n]$ is state vector, and $d(t)$ is an external bounded disturbance. The tracking errors are defined as

$$e_i = y_{ri} - x_i, \quad i = 1, \ldots, n, \tag{2}$$

where $\boldsymbol{y}_r = [y_{ri}, \ldots, y_{rn}]$ is the reference signal vector.

For approximating the unknown functions f and g, construct two interval type-2 fuzzy neural network (IT2-FNN) systems with the same fuzzy basic functions (FBFs) (from (24) and (25) of [1], ξ_f and ξ_g in (18) and (19) of [1] are equal to ξ):

$$\hat{f}(\boldsymbol{x}, \tau|\theta_f, \boldsymbol{m}, \boldsymbol{\sigma}) = \xi^{\mathrm{T}}(\boldsymbol{x}, \tau, \boldsymbol{m}, \boldsymbol{\sigma})\theta_f, \tag{3}$$

$$\hat{g}(\boldsymbol{x}, \tau|\theta_g, \boldsymbol{m}, \boldsymbol{\sigma}) = \xi^{\mathrm{T}}(\boldsymbol{x}, \tau, \boldsymbol{m}, \boldsymbol{\sigma})\theta_g. \tag{4}$$

And give an indirect fuzzy controller scheme:

$$u = \frac{1}{\hat{g}(\boldsymbol{x}, \tau|\theta_g, \boldsymbol{m}, \boldsymbol{\sigma})}\left(-\hat{f}(\boldsymbol{x}, \tau|\theta_f, \boldsymbol{m}, \boldsymbol{\sigma}) + y_r^{(n)} + \boldsymbol{k}^{\mathrm{T}}\boldsymbol{e} - u_a\right), \tag{5}$$

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where real vector $\boldsymbol{k} = [k_1,\ldots,k_n]$ is chosen such that all roots of the polynomial $p(s) = s^n + k_n s^{n-1} + \cdots + k_1$ are in the open left half side of s-plane, $e = [e_1,\ldots,e_n]$ is the error vector, u_a is a robust compensator. Then Theorem 1 in [1] shows that under controller (5) with the robust compensator (29) and adaptive laws (30)–(32) and (33) in [1], the H^∞ tracking performance of the system (1) can be achieved and all the variables of the closed-loop system are bounded.

However, we believe that using the same FBFs with identical adjusting parameters $\boldsymbol{m}$ and $\boldsymbol{\sigma}$ in two IT2-FNN systems $\hat{f}$ and $\hat{g}$ is incorrect, because $\hat{f}$ and $\hat{g}$ are used to approximate two different objective functions. So the result of Theorem 1 in [1] is inappropriate. For solving this problem, in this paper, we redefine some variables of IT2-FNN and correct several formular mistakes in [1] firstly. And then, we present novel parameter adaptive laws for IT2-FNN systems.

2. Preliminary

For the convenience of analysis, we must redefine some variables and give correct expressions of IT2-FNN systems. According to [2,3], a Gaussian type interval type-2 fuzzy set (IT2-FS) can be expressed as

$$\mu_j(x_i) = \exp\left[-\frac{1}{2}\frac{(x_i - m_{ij})^2}{(\sigma_{ij})^2}\right], \quad m_{ij} \in [\underline{m}_{ij}, \overline{m}_{ij}], \quad i = 1,\ldots,n,\ j = 1,\ldots,s, \tag{6}$$

where m_{ij} and σ_{ij} are, respectively, the mean and the standard deviation of the Gaussian function in the jth term of the ith input x_i, s is the number of the linguistic values with respect to x_i. Let $\boldsymbol{m} = [m_{11},\ldots,m_{1s},\ldots,m_{n1},\ldots,m_{ns}]$, $\boldsymbol{\sigma} = [\sigma_{11},\ldots,\sigma_{1s},\ldots,\sigma_{n1},\ldots,\sigma_{ns}]$. Then the IT2-FNN can be expressed as follows:

$$y(\boldsymbol{x}, \boldsymbol{m}, \boldsymbol{\sigma}|\theta) = \xi^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{m}, \boldsymbol{\sigma})\theta. \tag{7}$$

So according to (7), we can redefine $\hat{f}$ and $\hat{g}$ as

$$\hat{f}(\boldsymbol{x}, \tau|\boldsymbol{\theta}_f, \boldsymbol{m}_f, \boldsymbol{\sigma}_f) = \xi_f^{\mathrm{T}}(\boldsymbol{x}, \tau, \boldsymbol{m}_f, \boldsymbol{\sigma}_f)\boldsymbol{\theta}_f, \tag{8}$$

$$\hat{g}(\boldsymbol{x}, \tau|\boldsymbol{\theta}_g, \boldsymbol{m}_g, \boldsymbol{\sigma}_g) = \xi_g^{\mathrm{T}}(\boldsymbol{x}, \tau, \boldsymbol{m}_g, \boldsymbol{\sigma}_g)\boldsymbol{\theta}_g, \tag{9}$$

where subscripts f and g are used to distinguish the parameters of two different IT2-FNN systems.

3. Discussion

Define parameter errors as $\tilde{\boldsymbol{\theta}}_f = \boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*, \tilde{\boldsymbol{\theta}}_g = \boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*, \tilde{\boldsymbol{m}}_f = \boldsymbol{m}_f - \boldsymbol{m}_f^*, \tilde{\boldsymbol{\sigma}}_f = \boldsymbol{\sigma}_f - \boldsymbol{\sigma}_f^*$, where optimal parameter vectors $\boldsymbol{\theta}_f^*, \boldsymbol{m}_f^*$ and $\boldsymbol{\sigma}_f^*$ are defined in (20)–(23) of [1]. Denoting $\hat{f}(\boldsymbol{x}, \tau|\boldsymbol{\theta}_f^*, \boldsymbol{m}_f^*, \boldsymbol{\sigma}_f^*)$ by $\hat{f}^*$ and making Taylor series expansion of $\hat{f}^*$, we obtain

$$\hat{f}^* = \hat{f} - \left.\frac{\partial \hat{f}}{\partial \boldsymbol{\theta}_f}\right|_{\boldsymbol{\theta}_f^* = \boldsymbol{\theta}_f} \tilde{\boldsymbol{\theta}}_f - \left.\frac{\partial \hat{f}}{\partial \boldsymbol{m}_f}\right|_{\boldsymbol{m}_f^* = \boldsymbol{m}_f} \tilde{\boldsymbol{m}}_f - \left.\frac{\partial \hat{f}}{\partial \boldsymbol{\sigma}_f}\right|_{\boldsymbol{\sigma}_f^* = \boldsymbol{\sigma}_f} \tilde{\boldsymbol{\sigma}}_f + \omega_f = \hat{f} - \xi_f^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_f - \tilde{\boldsymbol{m}}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} \boldsymbol{\theta}_f^* - \tilde{\boldsymbol{\sigma}}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}} \boldsymbol{\theta}_f^* + \omega_f, \tag{10}$$

where $\xi_{\boldsymbol{m}_f} = \partial \xi_f / \partial \boldsymbol{m}_f, \xi_{\boldsymbol{\sigma}_f} = \partial \xi_f / \partial \boldsymbol{\sigma}_f$, and ω_f is the higher order term of the Taylor series. So we get

$$\begin{aligned}\hat{f} - \hat{f}^* &= \xi_f^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_f + \tilde{\boldsymbol{m}}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} \boldsymbol{\theta}_f^* + \tilde{\boldsymbol{\sigma}}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}} \boldsymbol{\theta}_f^* + \omega_f = \xi_f^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_f + \tilde{\boldsymbol{m}}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} \boldsymbol{\theta}_f + \tilde{\boldsymbol{\sigma}}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}} \boldsymbol{\theta}_f - \tilde{\boldsymbol{m}}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_f - \tilde{\boldsymbol{\sigma}}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_f + \omega_f \\ &= \left(\xi_f^{\mathrm{T}} - \boldsymbol{m}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} - \boldsymbol{\sigma}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}}\right)\tilde{\boldsymbol{\theta}}_f + \left(\tilde{\boldsymbol{m}}_f \xi_{\boldsymbol{m}_f}^{\mathrm{T}} + \tilde{\boldsymbol{\sigma}}_f \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}}\right)\boldsymbol{\theta}_f + \left(\boldsymbol{m}_f^* \xi_{\boldsymbol{m}_f}^{\mathrm{T}} + \boldsymbol{\sigma}_f^* \xi_{\boldsymbol{\sigma}_f}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_f\right) + \omega_f.\end{aligned} \tag{11}$$

Similarly, denoting $\partial \xi_g / \partial \boldsymbol{m}_g$ and $\partial \xi_g / \partial \boldsymbol{\sigma}_g$ by $\xi_{\boldsymbol{m}_g}$ and $\xi_{\boldsymbol{\sigma}_g}$, respectively, we have

$$\hat{g} - \hat{g}^* = \left(\xi_g^{\mathrm{T}} - \boldsymbol{m}_g \xi_{\boldsymbol{m}_g}^{\mathrm{T}} - \boldsymbol{\sigma}_g \xi_{\boldsymbol{\sigma}_g}^{\mathrm{T}}\right)\tilde{\boldsymbol{\theta}}_g + \left(\tilde{\boldsymbol{m}}_g \xi_{\boldsymbol{m}_g}^{\mathrm{T}} + \tilde{\boldsymbol{\sigma}}_g \xi_{\boldsymbol{\sigma}_g}^{\mathrm{T}}\right)\boldsymbol{\theta}_g + \left(\boldsymbol{m}_g^* \xi_{\boldsymbol{m}_g}^{\mathrm{T}} + \boldsymbol{\sigma}_g^* \xi_{\boldsymbol{\sigma}_g}^{\mathrm{T}}\right)\tilde{\boldsymbol{\theta}}_g + \omega_g. \tag{12}$$

Different from (35) and (36) in [1], (11) and (12) use subscripts f and g to distinguish two different IT2-FNN systems, and add superscript T to correct the mistake in (35) and (36) of [1].

According to (1) and (5), the last term of error dynamics (27) in [1] is redundant, that is, the error dynamic must be rewritten into

$$\dot{\boldsymbol{e}} = A\boldsymbol{e} + B\left[(\hat{f} - f) + (\hat{g} - g)u - d\right] + Bu_a, \tag{13}$$

where $A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$.

Define optimal fuzzy approximation error as

$$\omega_{\mathrm{opt}} = (\hat{f}^* - f) + (\hat{g}^* - g)u. \tag{14}$$