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Short communication

New variable separation solutions and nonlinear phenomena for the (2+1)-dimensional modified Korteweg–de Vries equation

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ABSTRACT

Variable separation approach, which is a powerful approach in the linear science, has been successfully generalized to the nonlinear science as nonlinear variable separation methods. The (2 + 1)-dimensional modified Korteweg–de Vries (mKdV) equation is hereby investigated, and new variable separation solutions are obtained by the truncated Painlevé expansion method and the extended tanh-function method. By choosing appropriate functions for the solution involving three low-dimensional arbitrary functions, which is derived by the truncated Painlevé expansion method, two kinds of nonlinear phenomena, namely, dromion reconstruction and soliton fission phenomena, are discussed.

1. Introduction

As an important aspect of nonlinear science, soliton has achieved noticeable progress during the past several decades. Solitons, especially the (1 + 1)-dimensional ones, have been widely applied in many physics fields, such as fluid mechanics, condense matter physics, plasmas physics, nonlinear optics, etc. [1–12].

In this paper, the (2 + 1)-dimensional mKdV equation [13]

$$u_t + u_{xxx} - \frac{3u_x u_{xx}}{2u} + \frac{3u_x^3}{4u^2} + 2Av_x u + 2Au_x v = 0, \tag{1a}$$

$$u_x = v_y, \tag{1b}$$

with A being an arbitrary constant, will be investigated. In Ref. [13], Eq. (1) was proved to be Painlevé integrable, and two cases of similarity reductions and an exact variable separation solution by the multi-linear variable separation approach [14–17] were given out.

This paper is outlined as follows. We will search new variable separation solutions for the (2 + 1)-dimensional mKdV equation (1) in the following section. In Section 3, by choosing appropriate variable separation functions, dromion reconstruction and soliton fission phenomena will be discussed. And a brief summary is given in the last section.

2. New variable separation solutions

Eq. (1) can be rewritten as

$$4u^2u_t + 4u^2u_{xxx} - 6uu_xu_{xx} + 3u_x^3 + 8Au^3v_x + 8Au^2u_xv = 0, \tag{2a}$$

$$u_x = v_y. \tag{2b}$$

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According to the Painlevé analysis [18], we assume solutions of Eq. (2) have the form of the following generalized Laurent expansion:

$$u = \phi^{-\alpha} \sum_{j=0}^{\infty} u_j \phi^j, \tag{3}$$

$$v = \phi^{-\beta} \sum_{j=0}^{\infty} v_j \phi^j, \tag{4}$$

where α and β are both constants to be determined, and u_j, v_j and ϕ are all functions with respect to $\{x, y, t\}$. The leading-order analysis gives us that $\alpha = \beta = -2$. Then the truncated Painlevé expansion at the constant level term reads

$$u = u_0\phi^{-2} + u_1\phi^{-1} + u_2, \tag{5}$$

$$v = v_0\phi^{-2} + v_1\phi^{-1} + v_2, \tag{6}$$

with $\{u_2(x,y,t), v_2(x,y,t)\} = \{0, v_2(x,t)\}$ being the seed solution of Eq. (2) and $\{u(x,y,t), v(x,y,t)\}$ being another different solution.

Substituting Eqs. (5) and (6) with $u_2 = 0$ into Eq. (2), and making the coefficients of like powers of ϕ vanish, one can find that

$$u_0 = -\frac{3\phi_x\phi_y}{2A}, \quad v_0 = -\frac{3\phi_x^2}{2A}, \tag{7}$$

$$u_1 = \frac{3\phi_{xy}}{2A}, \quad v_1 = \frac{3\phi_{xx}}{2A}, \tag{8}$$

$$v_2 = -\frac{4\phi_x\phi_t - 3\phi_{xx}^2 + 4\phi_x\phi_{xxx}}{8A\phi_x^2}, \tag{9}$$

and $\phi(x,y,t)$ satisfies

$$2\phi_x^2[\phi_{yt} + \phi_{xxxy}] - \phi_{xy}[2\phi_x\phi_t - 3\phi_{xx}^2 + 2\phi_x\phi_{xxx}] - 3\phi_x\phi_{xx}\phi_{xxy} = 0, \tag{10}$$

$$\begin{aligned} &2\phi_x^2\phi_{xxxy}\left[\phi_x\phi_y\phi_{xxy} + 2\phi_x\phi_{xy}^2 - \phi_y\phi_{xx}\phi_{xy}\right] - \phi_x\phi_{xxy}\left[3\phi_x\phi_y\phi_{xx}\phi_{xxy} + 6\phi_x\phi_{xx}\phi_{xy}^2 - 6\phi_y\phi_{xx}^2\phi_{xy} + 2\phi_x\phi_y\phi_{xy}\phi_{xxx}\right] \\ &\quad + \phi_{xy}^2\left[4\phi_x^3\phi_{yt} - 4\phi_x^2\phi_t\phi_{xy} + 6\phi_x\phi_{xx}^2\phi_{xy} - 3\phi_y\phi_{xx}^3 - 4\phi_x^2\phi_{xy}\phi_{xxx} + 2\phi_x\phi_y\phi_{xx}\phi_{xxx}\right] = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &-4\phi_x^3\phi_{xy}^2\phi_{xxxxy} + 6\phi_x^3\phi_{xy}\phi_{xxy}\phi_{xxxy} - \phi_x\phi_{xxy}\left[3\phi_{xx}^2\phi_{xy}^2 + 3\phi_x^2\phi_{xxy}^2 - 4\phi_x\phi_t\phi_{xy}^2 - 4\phi_x\phi_{xy}^2\phi_{xxx}\right] \\ &\quad + 2\phi_{xy}^2\left[2\phi_x^2\phi_{xt}\phi_{xy} - 2\phi_x^3\phi_{xyt} - 2\phi_x\phi_t\phi_{xx}\phi_{xy} + 3\phi_{xx}^3\phi_{xy} - 5\phi_x\phi_{xx}\phi_{xy}\phi_{xxx} + 2\phi_x^2\phi_{xy}\phi_{xxxx}\right] = 0. \end{aligned} \tag{12}$$

One can easily seen from above that $\{u, v\}$ expressed by Eqs. (5) and (6) is a solution of Eq. (2) if and only if Eqs. (10)–(12) hold. What is more, it is straightforward to verify that Eqs. (10)–(12) have a solution of the form

$$\phi(x,y,t) = a_0 + a_1F(x,t) + a_2G(y) + a_3F(x,t)H(y), \tag{13}$$

where F, G and H are all arbitrary functions of the indicated arguments, and a_0, a_1, a_2 and a_3 are all arbitrary constants.

Thus, variable separation solution of the (2 + 1)-dimensional mKdV equation (2), which includes three arbitrary functions, can be given by

$$u = \frac{3[-a_2(a_1 + a_3H)G_y + a_3(a_0 + a_2G)H_y]F_x}{2A(a_0 + a_1F + a_2G + a_3FH)^2}, \tag{14}$$

$$v = -\frac{3(a_1 + a_3H)^2F_x^2}{2A(a_0 + a_1F + a_2G + a_3FH)^2} + \frac{3(a_1 + a_3H)F_{xx}}{2A(a_0 + a_1F + a_2G + a_3FH)} + \frac{3F_{xx}^2 - 4F_xF_t - 4F_xF_{xxx}}{8AF_x^2}. \tag{15}$$

Following the extended tanh-function method [19], we can assume that Eq. (2) have a solution with the following ansätz:

$$u(x,y,t) = a_0 + \sum_{j=1}^{M} \left\{a_{2j-1}[\phi(\omega)]^j + a_{2j}[\phi(\omega)]^{-j}\right\}, \tag{16}$$

$$v(x,y,t) = b_0 + \sum_{j=1}^{N} \left\{b_{2j-1}[\phi(\omega)]^j + b_{2j}[\phi(\omega)]^{-j}\right\}, \tag{17}$$

where $a_0 = a_0(x,y,t)$, $a_{2j-1} = a_{2j-1}(x,y,t)$, $a_{2j} = a_{2j}(x,y,t)$, $b_0 = b_0(x,y,t)$, $b_{2j-1} = b_{2j-1}(x,y,t)$, $b_{2j} = b_{2j}(x,y,t)$ and $\omega = \omega(x,y,t)$ are all functions of $\{x,y,t\}$ to be determined, and $\phi(\omega(x,y,t))$ is the solution of the following Riccati equation:

$$\frac{d\phi}{d\omega} = \sigma + \phi^2. \tag{18}$$