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Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions

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ABSTRACT

In this paper, we investigate the heat transfer of a viscous fluid flow over a stretching/shrinking sheet with a convective boundary condition. Based on the exact solutions of the momentum equations, which are valid for the whole Navier–Stokes equations, the energy equation ignoring viscous dissipation is solved exactly and the effects of the mass transfer parameter, the Prandtl number, and the wall stretching/shrinking parameter on the temperature profiles and wall heat flux are presented and discussed. The solution is given as an incomplete Gamma function. It is found the convective boundary conditions results in temperature slip at the wall and this temperature slip is greatly affected by the mass transfer parameter, the Prandtl number, and the wall stretching/shrinking parameters. The temperature profiles in the fluid are also quite different from the prescribed wall temperature cases.

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1. Introduction

The dynamics of fluid flow over a stretching or shrinking surface is important in many practical applications, such as extrusion of plastic sheets, paper production, glass blowing, metal spinning, drawing plastic films, to name just a few [1–3]. The quality of the final product depends on the rate of heat and mass transfer between the stretching surface and fluid flow [4]. Since the pioneering study by Sakiadis [5] on the boundary layer flow over a continuously stretching surface with a constant speed, many researchers have investigated various aspects of this problem, such as consideration of mass transfer, power-law variation of the stretching velocity and temperature, magnetic field, application to non-Newtonian fluids, and similarity solutions were obtained [6–28].

Not many studies have been done about the problem of a shrinking sheet where the velocity on the boundary is towards a fixed point. Miklavčič and Wang [29] studied the flow over a shrinking sheet with mass flux (suction or injection), which is an exact solution of the Navier–Stokes equations. This new type of shrinking sheet flow is essentially a backward flow as discussed by Goldstein [30]. Problems of boundary layer flow over shrinking sheets with mass transfer have been studied [31–35]. The stagnation flow over a shrinking sheet was investigated by Wang [36]. Hayat et al. [37] studied the magneto-hydrodynamic (MHD) and rotating flow over a permeable shrinking sheet.

However, heat transfer characteristics of the stretching/shrinking sheet problem have been restricted to two boundary conditions of either prescribed temperatures or heat flux at the wall in the published papers. Most recently, heat transfer

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problems for boundary layer flow concerning a convective boundary condition was investigated by Aziz for the Blasius flow [38]. Similar analysis was applied to the Blasius and Sakiadis flow with radiation effects [39]. Motivated by the above-mentioned investigations and applications, in this paper, we investigate heat transfer problem with a convective boundary condition for a viscous and incompressible fluid over a permeable (with mass flux) stretching/shrinking sheet in a quiescent fluid.

2. Basic equations and exact solutions

Consider the steady two-dimensional flow of a viscous and incompressible fluid on a continuously stretching or shrinking sheet with mass transfer in a stationary fluid. It is assumed that the velocity of the stretching sheet is $u_w(x) = bx$, where b is a real number. It is also assumed that constant mass transfer velocity is v_w with $v_w < 0$ for suction and $v_w > 0$ for injection, respectively. The ambient fluid temperature is a constant T_∞. The sheet surface temperature is maintained by convective heat transfer at a certain value T_w, which is to be determined later. The x-axis is along the stretching surface and the y-axis is perpendicular to it. With these assumptions the governing equations of this problem can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\nabla^2 v, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sigma\nabla^2 T, \tag{4}$$

where u and v are the velocity components along the x- and y- axes, p is the pressure, ρ is the density, υ is the kinematic viscosity of the fluid, σ is the thermal diffusivity of the fluid, and T is the fluid temperature. In the energy equation, the viscous dissipation term is neglected. The boundary conditions (BCs) of these equations are

$$\begin{aligned}&u=u_w(x)=bx,\quad v=v_w,\quad -k\frac{\partial T}{\partial y}=h(T_f-T_w)\quad \text{at}\quad y=0,\\&u=0,\quad T=T_\infty,\quad \text{as}\quad y\to\infty,\end{aligned} \tag{5}$$

where k is the thermal conductivity of the fluid and h is the convective heat transfer coefficient, v_w is the wall mass transfer velocity, and T_f is the convective fluid temperature below the moving sheet. We assume that Eqs. (1)–(4) subject to the BCs (5) admit the similarity solutions,

$$u=axf'(\eta),\quad v=-\sqrt{a\upsilon}f(\eta),\quad T-T_\infty=\theta(\eta)(T_f-T_\infty),\quad \eta=y\sqrt{a/\upsilon}, \tag{6}$$

where primes denote differentiation with respect to η and a is a positive constant. We also denote $u_r(x) = ax$ as a reference velocity for this problem and $u = u_r(x)f'(\eta)$. Using Eq. (3) and the boundary conditions (5), we obtain the following expression for the pressure p

$$p=p_0-\rho\frac{v^2}{2}+\rho\upsilon\frac{dv}{dy}, \tag{7}$$

where p_0 is the stagnation pressure. Substituting (6) and (7) into Eqs. (2) and (4), we obtain the following ordinary differential equations

$$f'''+ff''-f'^2=0, \tag{8}$$

$$\theta''+\mathrm{Pr}f\theta'=0, \tag{9}$$

subject to the BCs

$$\begin{aligned}&f(0)=s,\quad f'(0)=b/a=\alpha,\quad \theta'(0)=-\gamma[1-\theta(0)],\\&f'(\infty)=0,\quad \theta(\infty)=0,\end{aligned} \tag{10}$$

where s is the mass transfer parameter with $s > 0$ for mass suction and $s < 0$ for mass injection, respectively. Also $\alpha > 0$ is for wall stretching and $\alpha < 0$ is for shrinking, respectively. Pr is the Prandtl number of the fluid with $\mathrm{Pr} = \upsilon/\sigma$. γ is the equivalent Biot number for this problem defined as $\gamma=\frac{h}{k}\sqrt{\frac{\upsilon}{a}}$, which is always positive.

There exists an exact solution for Eq. (8) together with the boundary conditions (10) as follows:

$$f(\eta)=\beta+(s-\beta)e^{-\beta\eta}=\beta-\frac{\alpha}{\beta}e^{-\beta\eta} \tag{11}$$

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