Commun Nonlinear Sci Numer Simulat 16 (2011) 776–786

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Mathematics analysis and chaos in an ecological model with an impulsive control strategy

Hengguo Yu [a,*], Shouming Zhong [a,b], Ravi P. Agarwal [c,d]

[a] School of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, China
[b] Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, China
[c] Department of Mathematical Science, Florida Institute of Technology, Melbourne, FL 32901, USA
[d] Mathematics and Statistics Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

ARTICLE INFO

Article history:
Available online 24 April 2010

Keywords:
Impulsive control strategy
The largest Lyapunov exponent
Locally asymptotically stable
Complex dynamics
Periodic solution
Chaotic behaviour

ABSTRACT

In this paper, on the basis of the theories and methods of ecology and ordinary differential equation, an ecological model with an impulsive control strategy is established. By using the theories of impulsive equation, small amplitude perturbation skills and comparison technique, we get the condition which guarantees the global asymptotical stability of the lowest-level prey and mid-level predator eradication periodic solution. It is proved that the system is permanent. Further, influences of the impulsive perturbation on the inherent oscillation are studied numerically, which shows rich dynamics, such as period-doubling bifurcation, period-halving bifurcation, chaotic band, narrow or wide periodic window, chaotic crises,etc. Moreover, the computation of the largest Lyapunov exponent demonstrates the chaotic dynamic behavior of the model. At the same time, we investigate the qualitative nature of strange attractor by using Fourier spectra. All these results may be useful for study of the dynamic complexity of ecosystems.

1. Introduction

The research of the last two decades demonstrates that very complex dynamics can arise in continuous time food chain models with three or more species [1–4], while similar results are obtained for multi-species food models with specialist and generalist top-predators [5–7].

Many evolution processes are characterized by the fact at certain moments of time when they experience a change of state abruptly. These processes are subject to short-term perturbations whose duration is negligible in comparison with the duration of the process. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulse. It is well known that many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics and frequency modulate systems do exhibit impulsive effects. Thus impulsive differential equations, differential equations involving impulsive effects, appear as a natural description of observed evolution phenomena of several real world problems [8–10]. Furthermore, the paper [11] presents a new approach via variational methods and critical point theory to obtain the existence of solutions to impulsive problems and the paper [12] has proved that there exists Li-York chaos in the system with impacts. The field of research of chaotic impulsive differential equations about biological and chemical control seems to be a new growing interesting area in the recent years, which some scholars have paid attention to [13–29].

In this paper, we consider a three-species ecological model with an impulsive control strategy. The model can be described by the following differential equations:

* Corresponding author. Tel.: +86 028 13699440836.
E-mail addresses: yuhengguo5340@163.com (H. Yu), zhongsm@uestc.edu.cn (S. Zhong), agarwal@fit.edu (R.P. Agarwal).

1007-5704/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2010.04.017

$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= r_1x(t)\left(1-\frac{x(t)}{k_1}\right)-\frac{a_1x(t)y(t)}{b_1+x(t)}-\frac{a_2x(t)z(t)}{b_2+x(t)} \\ \frac{dy(t)}{dt} &= r_2y(t)\left(1-\frac{y(t)}{k_2}\right)+\frac{e_1a_1x(t)y(t)}{b_1+x(t)}-\frac{a_3y(t)z(t)}{b_3+y(t)} \\ \frac{dz(t)}{dt} &= \frac{a_2e_2x(t)z(t)}{b_2+x(t)}+\frac{a_3e_3y(t)z(t)}{b_3+y(t)}-mz(t) \end{aligned}\right\} & t\neq nT, \\ \left.\begin{aligned} \Delta x(t) &= 0 \\ \Delta y(t) &= 0 \\ \Delta z(t) &= p \end{aligned}\right\} & t=nT, \end{cases} \tag{1.1}$$

where $x(t)$,$y(t)$,$z(t)$ are the densities of one prey and two predators at time t, respectively, and $\Delta x(t) = x(t^+)-x(t)$, $\Delta y(t) = y(t^+)-y(t)$, $\Delta z(t) = z(t^+)-z(t)$. $r_i(i = 1,2)$ are the intrinsic growth rate, $a_i(i = 1,2,3)$ and b_i, $(i = 1,2)$ measure the efficience of the prey in evading a predator attack, b_3 has similar meaning as that of b_i. $e_i(i = 1,2,3)$ denote the efficiency with which resources are converted to new consumers, k_i, $(i = 1,2)$ are carrying capacity in the absence of predator, $m$is the mortality rates for the predator, And then T is the period of the impulsive effect, $n \in N$, N is the set of all non-negative integers, $p > 0$ is the release amount of predator at $t = nT$.

In this paper, we firstly prove that the lowest-level prey and mid-level predator eradication periodic solution is globally.asymptotically stable and the system is permanent. Secondly, by using the numerical simulation, we investigate the influences on the inherent oscillation caused by the impulsive perturbations. Finally, the computation of the largest Lyapunov exponent demonstrates the chaotic dynamic behavior of the model and the qualitative natures of the strange attractors are studied by using Fourier spectra.

2. Mathematical analysis

Let $R_+ = [0,\infty)$, $R^3_+ = \{X \in R^3 \mid X \geqslant 0\}$. Denote $f = (f_1,f_2,f_3)$ the map defined by the right hand of the first second third equation of system (1.1). Let $V: R_+ \times R^3_+ \rightarrow R_+$, then V is said to belong to class V_0 exists.

(1) V is continuous in $(nT,(n+1)T] \times R^3_+$, and for each $X \in R^3_+$, $n \in N$, $\lim_{(t,y)\rightarrow(nT^+,X)} V(t,y) = V(nT^+,X)$ exists.
(2) V is locally Lipschitzian in X.

Definition 2.1. Let $V \in V_0$, then for $(t,x) \in (nT,(n+1)T] \times R^3_+$, the upper right derivative of $V(t,X)$ with respect to the impulsive differential system (1.1) is defined as

$$D^+V(t,X) = \lim_{h\rightarrow 0^+} \sup \frac{1}{h}[V(t+h,X+hf(t,x))-V(t,X)]$$

The solution of system (1.1) is a piecewise continuous function $X: R_+ \rightarrow R^3_+$, $X(t)$ is continuous on $(nT, (n + 1)T]$, $n \in N$ and $X(nT^+) = \lim_{t\rightarrow nT^+} X(t)$ exists. The smoothness properties of f guarantee the global existence and uniqueness of solution of system (1.1), for the details see book [8–10].

Definition 2.2. System (1.1) is said to be permanent if there exists a compact $\Omega \subset \text{int}R^3_+$ such that every solution $(x(t),y(t),z(t))$ of system (1.1) will eventually enter and remain in the region Ω.

The following lemma is obvious.

Lemma 2.1. *Let $X(t)$ is a solution of system (1.1) with $X(0^+) \geqslant 0$, then $X(t) \geqslant 0$ for all $t \geqslant 0$. And further $X(t) > 0$, $t > 0$ if $X(0^+) > 0$.*

We will use an important comparison theorem on impulsive differential equation.

Lemma 2.2 (Lakshmikantham 1989). *Suppose $V \in V_0$. Assume that*

$$\begin{cases} D^+V(t,X) \leqslant g(t,V(t,X)) & t\neq nT, \\ V(t,X(t^+)) \leqslant \psi_n(V(t,X)) & t=nT, \end{cases} \tag{2.1}$$

where $g: R_+ \times R_+ \rightarrow R$ is continuous in $(nT, (n + 1)T] \times R_+$ and for $u \in R_+, n \in N$, $\lim_{(t,v)\rightarrow(nT^+,u)} g(t,v) = g(nT^+,u)$ exists, $\psi_n: R_+ \rightarrow R_+$ is non-decreasing. Let $r(t)$ be maximal solution of the scalar impulsive differential equation

$$\begin{cases} \frac{du(t)}{dt} = g(t,u(t)) & t\neq nT, \\ u(t^+) = \psi_n(u(t)) & t=nT, \\ u(0^+) = u_0, \end{cases} \tag{2.2}$$

existing on $[0,\infty)$. Then $V(0^+,X_0) \leqslant u_0$, implies that $V(t,X(t)) \leqslant r(t), t \geqslant 0$, where $X(t)$ is any solution of system (1.1).

Finally, we give some basic properties about the following subsystems of system (1.1)