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Three-species food web model with impulsive control strategy and chaos

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ABSTRACT

A three-species ecological model with impulsive control strategy is developed using the theory and methods of ecology and ordinary differential equation. Conditions for extinction of the system are given based on the theory of impulsive equation and small amplitude perturbation. Using comparison involving multiple Lyapunov functions, the system is shown to be permanent. Further, the influence of the impulsive perturbation on the inherent oscillation are studied numerically and is found to depict rich dynamics, such as the period-doubling bifurcation, the period-halving bifurcation, a chaotic band, a narrow or wide periodic window, and chaotic crises. In addition, the largest Lyapunov exponent is computed. This computation demonstrates the chaotic dynamic behavior of the model. The qualitative nature of concerned strange attractors is also investigated through their computed Fourier spectra. The foregoing results have the potential to be useful for the study of the dynamic complexity of ecosystems.

1. Introduction

The research of the last two decades demonstrated that very complex dynamics could arise in continuous time food chain models with three or more species [1–4]. Similar results were obtained for multi-species food web models [5–7].

Many evolution processes are characterized by the fact that at certain moments these experience an abrupt change of state. Such a process is subjected to a short-term perturbation whose duration is negligible in comparison with the duration of the process. Consequently, it is natural to assume that this perturbation acts instantaneously, viz., in the form of an impulse. It is well known that many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics, and frequency modulated systems do exhibit impulsive effects. Thus impulsive differential equations/differential equations involving impulsive effects, appear as a natural mathematical model for observed evolution phenomena of several real world problems [8,9]. The chaotic impulsive differential equations concerning biological control appears to be a new growing research area in recent years [10–22]. In this paper, we consider a three-species ecological model with impulsive control strategy. The model is described by the following differential equations:

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$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= r_1x(t)\left(1-\frac{x(t)}{k_1}-\frac{h_1y(t)}{k_1}\right)-\frac{a_1x(t)z(t)}{b+x(t)+dy(t)+cz(t)},\\ \frac{dy(t)}{dt} &= r_2y(t)\left(1-\frac{y(t)}{k_2}-\frac{h_2x(t)}{k_2}\right)-\frac{a_2y(t)z(t)}{b+x(t)+dy(t)+cz(t)},\\ \frac{dz(t)}{dt} &= \frac{a_1e_1x(t)z(t)}{b+x(t)+dy(t)+cz(t)}+\frac{a_2e_2y(t)z(t)}{b+x(t)+dy(t)+cz(t)}-mz(t), \end{aligned}\right\} & t\neq nT,\ t\neq(n+l-1)T,\\ \left.\begin{aligned} \Delta x(t) &= -\delta_1x(t),\\ \Delta y(t) &= -\delta_2y(t),\\ \Delta z(t) &= -\delta_3z(t), \end{aligned}\right\} & t=(n+l-1)T,\\ \left.\begin{aligned} \Delta x(t) &= 0,\\ \Delta y(t) &= 0,\\ \Delta z(t) &= p, \end{aligned}\right\} & t=nT, \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$, $z(t)$ are the densities of the two preys and a predator at time t, respectively, $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, $\Delta z(t) = z(t^+) - z(t)$. The parameter $0 < l < 1$ is used to describe the time intervals in the pulsed use of controls. r_i $(i = 1,2)$ are the intrinsic growth rates, a_i $(i = 1,2)$ and b are saturating functional response parameters, e_i $(i = 1,2)$ denote the efficiency with which resources are converted to new consumers, d is the relative preference of predator on prey y over prey x. The parameters h_i $(i = 1,2)$ are the interspecific competition rates between the two preys species, while m is the mortality rate for the predator and $c\,z(t)$ measures the mutual interference in predators. Further, the parameter T is the time period of the impulsive effect. The integer $n \in N$, where N is the set of all non-negative integers, $p > 0$ is the release amount of predator at $t = nT$, δ_i $(i = 1,2,3)(0 \leqslant \delta_i \leqslant 1)$ represent the fractions of prey–predator that dies at time $t = (n + l - 1)T$.

Here we first study, in general, the effect of impulsive perturbations, establish conditions for extinction, and obtain the condition of permanence of the system (1.1). We then, using a numerical method, study the influence on the inherent oscillations caused by the impulsive perturbations. Finally, we compute the largest Lyapunov exponent that also demonstrates the chaotic dynamic behavior of the model and the qualitative nature of strange attractors. This computation is investigated using Fourier spectra.

2. Mathematical analysis

Let $R_+ = [0,\infty)$, $R^3_+ = \left\{X = (x(t), y(t), z(t)) \in R^3 | X \geqslant 0\right\}$. Denote by $f = (f_1, f_2, f_3)$ the map defined by the right-hand side expressions of the first, second, and third equations of the system (1.1). If $V : R_+ \times R^3_+ \to R_+$, then V is said to belong to class V_0.

(1) V is continuous in $((n-1)T, (n+l-1)T] \times R^3_+$ and $((n+l-1)T, nT] \times R^3_+$. For each $X \in R^3_+, n \in N, \lim_{(t,\mu)\to((n+l-1)T^+,X)} V(t,\mu) = V((n+l-1)T^+, X)$ and $\lim_{(t,\mu)\to(nT^+,X)} V(t,\mu) = V(nT^+, X)$ exist.
(2) V is locally Lipschitzian in X.

Definition 2.1. Let $V \in V_0$, then for $((n-1)T, (n+l-1)T] \times R^3_+$ and $((n+l-1)T, nT] \times R^3_+$, the upper right derivative of $V(t,X)$ with respect to the impulsive differential system (1.1) is defined as

$$D^+V(t,X) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(t+h, X+hf(t,X)) - V(t,X)].$$

The solution of the system (1.1) is a piecewise continuous function $X : R_+ \to R^3_+$, $X(t)$ is continuous on $((n-1)T, (n+l-1)T) \cup ((n+l-1)T, nT), n \in N, 0 < l < 1, X((n+l-1)T^+) = \lim_{t\to(n+l-1)T^+} X(t)$ and $X(nT^+) = \lim_{t\to nT^+} X(t)$ exist. The smoothness properties of f guarantee the global existence and uniqueness of solution of the system (1.1). For details see [8,9].

Definition 2.2. The system (1.1) is said to be permanent if there exists a compact region $\Omega \subset \mathrm{int}R^3_+$ such that every solution $(x(t), y(t), z(t))$ of the system (1.1) will eventually enter and remain in the region Ω.

We present the following lemmas without proof as these are straightforward.

Lemma 2.1. *Let $X(t)$ be a solution of the system (1.1) with $X(0^+) \geqslant 0$, then $X(t) \geqslant 0$ for all $t \geqslant 0$. Further $X(t) > 0, t > 0$ if $X(0^+) > 0$.*

Lemma 2.2. *There exists a constant M such that $x(t) \leqslant M, y(t) \leqslant M$ and $z(t) \leqslant M$ for each solution $(x(t), y(t), z(t))$ of the system (1.1) with all sufficiently large t.*

We will use later an important comparison theorem on impulsive differential equation.

Lemma 2.3 (*Bainov and Simeonov [9]*). *Suppose $V \in V_0$. Assume that*

$$\begin{cases} D^+V(t,X) \leqslant g(t, V(t,X)), & t \neq nT,\ t \neq (n+l-1)T,\\ V(t, X(t^+)) \leqslant \psi^1_n(V(t,X)), & t = (n+l-1)T,\\ V(t, X(t^+)) \leqslant \psi^2_n(V(t,X)), & t = nT, \end{cases} \tag{2.1}$$