Commun Nonlinear Sci Numer Simulat 15 (2010) 1640–1647

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Linear observer based projective synchronization in delay Rössler system

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ARTICLE INFO

Article history:
Received 16 December 2008
Received in revised form 26 March 2009
Accepted 18 June 2009
Available online 23 June 2009

PACS:
05.45.Xt
05.45.Pq
05.45.Gg

Keywords:
Projective synchronization
Delay Rössler
Linear observer
Modulated time delay

ABSTRACT

A new type of linear observer based projective, projective anticipating and projective lag synchronization of time-delayed Rössler system is studied. Along with this, the approach arbitrarily scales a drive system attractor and hence a similar chaotic attractor of any desired scale can be realized with the help of a synchronizing scaling factor. A scalar synchronizing output is considered where the output equation includes both the delay and non-delay terms of the nonlinear function. The condition for synchronization is derived analytically and the values of the coupling parameters are obtained. Analytical results are verified through numerical investigation and the effect of modulated time delay in the method is discussed. An important aspect of this method is that it does not require the computation of conditional Lyapunov exponents for the verification of synchronization.

1. Introduction

In recent years, synchronization of chaotic systems has been an active area of research in various fields of science such as nonlinear circuits, biophysical systems, chemical reactions, ecological systems, laser systems and many more [1–8]. Many techniques has been developed and synchronization effect of chaotic nonlinear dynamical systems has been applied to secure communications successfully. After the pioneering work of Pecora and Carroll [9], different types of synchronization, namely, complete, generalized, lag and phase synchronization has been studied theoretically and experimentally [10–14]. Other than these, anticipatory synchronization has also led to extensive research in the field of chaotic systems [15,16]. In 1998, Gonzalez-Miranda [17] observed that when chaotic systems exhibits invariance properties under a special type of continuous transformations amplification and displacement of the attractor occurs. In 1999, Mainieri and Rehacek [18] observed a new type of synchronization in coupled partially linear chaotic systems where drive and response systems can synchronize with a scaling factor α and it is a proportional relation. In partially linear chaotic systems, such as Lorenz system, it was observed that the states of the coupled subsystems could be synchronized up to a constant ratio. This type of synchronization as a special form of generalized synchronization is referred to as projective synchronization [19]. Special cases of projective synchronization are complete and anti-phase synchronization for the scaling factor equal to 1.0 and −1.0, respectively. This type of synchronization results from the partial linearity of the coupled chaotic systems and has become a unique feature of partially linear systems. So far little attention has been paid on projective synchronization and even lesser

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1007-5704/$ - see front matter © 2009 Published by Elsevier B.V.
doi:10.1016/j.cnsns.2009.06.019

when a linear observer based method is used to obtain projective synchronization [20,21]. Projective synchronization along with a linear observer gives a technique that can be applied to a wide class of chaotic and hyper-chaotic systems

In this communication, we analytically and numerically analyze the existence of projective, projective lag and projective anticipatory synchronization via linear observer based method in a time delay Rössler system. The possibility of projective synchronization in high dimensional nonlinear systems has been speculated since it was observed in three dimensional systems. Time delay systems being infinite dimensional are interesting topic of research and the dimension of their chaotic dynamics can be increased by increasing the delay time [22–24]. Chaotic time delay systems are helpful in secure communication and encryption schemes. Due to finite signal transmission times, switching speeds and memory effects with both single and multiple time delays are ubiquitous in nature. Delay systems thus can be an interesting topic in synchronization and so far few works has been done where a linear observer is taken into account along with it.

In three dimensional systems the condition of projective synchronization is that the trace of the Jacobian matrix of the linear part of the system must be negative i.e. the matrix must be a full rank matrix. Previously, a generic method based on Lyapunov stability theory was applied to obtain projective, projective lag and projective anticipatory synchronization of time-delayed chaotic systems. The attractor of the response system being the scaled replica of the drive system, possess the same topological characteristics, such as, Lyapunov exponents and fractal dimensions as that of the drive system. The performance of projective synchronization can be selected and manipulated by controlling the scaling factor. Such manipulations can also be extended to non partial linear systems [25]. The method described here is highly systematic and rigorous. One of the problems that arises in synchronization methods during the coupling of the drive and response systems is that the output signal from the drive system needs to be properly placed in the response system else no synchronization occur. There is no proper way of identifying the exact position of the signal in the response system. This method can easily overcome this problem and hence is much generalized and exact. Moreover one of the important conditions of indicating the onset of synchronization is that the largest conditional Lyapunov exponent should tend towards a negative value. This technique has the advantage that it does not require computation of conditional Lyapunov exponents[20]. Thus, the method can be applied to several hyper-chaotic time delay systems such as Mackey-Glass [26] and delayed cellular neural networks [27]. Very recently Hu et al. [28] studied the projective synchronization on drive response dynamical networks considering coupled Lorenz chaotic systems.

2. Approach to projective synchronization

Two dynamical systems with state vectors $x(t)$ and $\hat{x}(t)$, are in projective synchronization when for an initial condition $\hat{x}(0)$, there is a constant α such that [18]

$$\hat{x}(t) = \alpha x(t) \quad as\ t \to \infty$$

We focus on the class of time delay systems defined by the following state and output equation

$$\dot{x}(t) = Ax(t) + Bf(x) + g(x(t-\tau)) + C \tag{1}$$

$$s(t) = Bf(x) + g(x(t-\tau)) + kx(t) \tag{2}$$

where, $x(t) \in R^{n\times 1}$ is the state vector, $k \in R^{n\times n}$ is the coupling strength matrix and $A \in R^{n\times n}$, $C \in R^{n\times 1}$ and $B \in R^{n\times n}$ are constant matrices.

$s(t)$ is the scalar synchronizing output and f and g are nonlinear functions without and with delay, respectively. The approach enables to reconstruct state vector $x(t)$ by a response system using a scalar function as shown. As a result we need not make a copy of the drive system in order to achieve synchronization. If the scalar synchronization output is collected after a certain time given by τ_p, then it is the time delay of the observer prediction. The state and output equations of the response system are defined as,

$$\dot{\hat{x}} = A\hat{x}(t) + (\alpha s(t-\tau_p) - \hat{s}(t)) + \alpha C \tag{3}$$

$$\hat{s}(t) = k\hat{x}(t) \tag{4}$$

$\hat{s}(t)$ is the observer prediction of $s(t)$. Let $e(t) = \hat{x}(t) - \alpha x(t-\tau_p)$ be the projective synchronization error. The error system is given by

$$\dot{e}(t) = \dot{\hat{x}} - \alpha\dot{x}(t-\tau_p) = [A-k]e(t) \tag{5}$$

For any scaling factor α, the driver system and the linear observer synchronize if the controllability matrix $[k, Ak, A^2k \ldots A^{n-1}k]$ is full rank.

If $\tau_p = 0$ then one obtains exact projective synchronization and it is complete and anti-phase for $\alpha = 1$ and $\alpha = -1$, respectively.

If $\tau_p > 0$ then it shows projective lag synchronization.

If $\tau_p < 0$ then it shows projective anticipatory synchronization.