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Multiperiodicity analysis and numerical simulation of discrete-time transiently chaotic non-autonomous neural networks with time-varying delays ☆

Zhenkun Huang [a,*], Sannay Mohamod [b], Honghua Bin [a]

[a] *School of Sciences, Jimei University, Xiamen, 361021 Fujian, China*
[b] *Department of Mathematics, Faculty of Science, Universiti Brunei Darussalam, Gadong BE 1410, Brunei Darussalam*

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A B S T R A C T

In this paper, we investigate multiperiodicity analysis of discrete-time transiently chaotic neural networks, i.e., the coexistence and exponential stability of multiple periodic sequence solutions. By using analytic property of activation functions and Schauder's fixed point theorem, we attain the coexistence of 2^N periodic sequence solutions. Meanwhile, some new and simple criteria are derived for the networks to converge exponentially toward 2^N periodic sequence solutions. Our results are new and complement existing ones in the literature. Finally, computer numerical simulations are performed to illustrate multiperiodicity of discrete-time transiently chaotic neural networks.

1. Introduction

In recent years, the dynamical behaviors of neural networks with or without delays have been widely investigated and applied to various information processing problems such as associative memories, combinational optimization, pattern classification and so on, e.g., [1–3,7,12,16,19,20,27,28,30,32]. Moreover, due to their promise and power in the variety of important applications, the stability of equilibrium points and the existence of periodic orbits together with global exponential stability of discrete-time neural networks have also been widely discussed, we can refer to [4–9,12–18,20–26,29,31]. As a kind of well-known neural model, the transiently chaotic neural networks with chaotic simulated annealing have been shown to be a promising tool for combinational optimization [11,12,33]. Therefore, a variety of competing results have been

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* Corresponding author. Tel.: +86 592 6182690.
E-mail address: hzk974226@jmu.edu.cn (Z. Huang).

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accumulated in the literature concerning the global asymptotical stability criterion for equilibrium points, periodic attractor and chaotic criterion of transiently chaotic neural networks [10–14,34].

Meanwhile, when the neural networks are designed for a memory system with a large information capacity, the addressable memories or patterns are stored as stable equilibria or stable periodic orbits. Thus, in designing associative memories, it is an important task to discuss convergence analysis and coexistence of multiple equilibria or multiple periodic orbits of neural networks [3,4,6,7,19,39,40].

For discrete-time transiently chaotic neural networks, the existence of equilibrium and convergence theorem by LaSalle's invariance principle have been attained recently in [13]. Later, authors in [14] furthered to consider nonautonomous (periodic) case and derived some sufficient conditions for the existence of periodic solution by using the topological degree theory. However, there are too few results reported for multiperiodicity of discrete-time transiently chaotic neural networks. Stimulated by [3,6,7,14,19,31,35], we should consider the following discrete-time transiently chaotic neural networks:

$$\begin{cases} y^i(n+1) = k(n)y^i(n) + \sum\limits_{j=1}^{N} w_{ij}(n)x^j\big(y^j(n-\upsilon(n))\big) + a_i(n) - w_{ii}(n)a_{0i}(n), \\ x^j\big(y^j(n)\big) = \frac{1}{1+\exp\left(-y^j(n)/\epsilon\right)}, \quad n = 0,1,2,\ldots \end{cases} \tag{1.1}$$

where $i \in \mathscr{N} := \{1,2,\ldots,N\}$, the nonnegative integer $\upsilon(n)$ denotes the time-varying delay satisfying $0 \leqslant \upsilon(n) \leqslant \upsilon$. It is obvious that system (1.1) includes discrete-time neural networks considered by [14,34] as its special cases when $\upsilon(n) \equiv 0$. Our main aim is to study multiperiodicity of above discrete-time neural networks, that is, coexistence of 2^N periodic sequence solutions and their exponential stability. Our results are completely different from most of the existing results in [10–14,34]. Particularly, when system (1.1) degenerates into autonomous system, our results can extend transiently chaotic neural networks considered in [34] to multistability analysis (e.g., coexistence and stability of multiple equilibria).

The rest of this paper is organized as follows. In Section 2, we shall make some preparations by giving some notations and a basic lemma. Meanwhile, we propose some new assumptions for discrete-time neural networks by using analytic property of activation functions. In Section 3, by using Schauder's fixed point theorem, we establish some new criteria for the existence and exponential stability of 2^N periodic sequence solutions. Finally, computer numerical simulations are presented to illustrate our results.

2. Preliminary

Denote $\mathbb{Z}$ as the set of all integers and $\mathbb{Z}^{[a,b]} := \mathbb{Z} \cap [a,b]$, where $a,b \in \mathbb{Z}$ and $a \leqslant b$. When $b = +\infty$, we simply denote $\mathbb{Z}^{[a]} := \mathbb{Z}^{[a,+\infty]} = \{a, a+1, \ldots\}$. Then the set of all nonnegative integers can be denoted by $\mathbb{Z}^{[0]}$. Let $S(\mathbb{Z}^{[-\upsilon,0]}, \mathbb{R}^N)$ be the Banach space of all functions $\psi(s) = (\psi^1(s), \psi^2(s), \ldots, \psi^N(s))$ mapping $\mathbb{Z}^{[-\upsilon,0]}$ into $\mathbb{R}^N$ with norm defined by $\|\psi\| = \max_{i\in\mathscr{N}}\left\{\max_{s\in\mathbb{Z}^{[-\upsilon,0]}}|\psi^i(s)|\right\}$. For any given initial condition $\psi \in S(\mathbb{Z}^{[-\upsilon,0]}, \mathbb{R}^N)$, we denote by $\{y(n;\psi)\}$ the sequence solution of system (1.1). Let ℓ be a given nonnegative integer and $y_\ell \in S(\mathbb{Z}^{[-\upsilon,0]}, \mathbb{R}^N)$ be defined by $y_\ell(s) = y(\ell+s), s \in \mathbb{Z}^{[-\upsilon,0]}$. Then we can rewrite the initial condition as $y_0 = \psi \in S(\mathbb{Z}^{[-\upsilon,0]}, \mathbb{R}^N)$. Let $(\mathscr{P}, \|\cdot\|)$ be the space of all θ-periodic sequences defined on $\mathbb{Z}$ with norm $\|x\| = \max_{n\in\mathbb{Z}^{[0,\theta]}}|x(n)|$ for any $x = \{x(n)\} \in \mathscr{P}$. Then $(\mathscr{P}, \|\cdot\|)$ is a Banach space. Throughout this paper, for each $i,j \in \mathscr{N}$, we always suppose that

- (H_1) $\{k(n)\}$, $\{w_{ij}(n)\}$, $\{\upsilon(n)\}$, $\{a_i(n)\}$ and $\{a_{0i}(n)\}$ are all θ-periodic sequences; Moreover, $k(n) > 0$, $\min_{n\in\mathbb{Z}^{[0,\theta]}}\{w_{ii}(n)\} > 0$ and $K := \max_{n\in\mathbb{Z}^{[0,\theta]}}\{k(n)\} < 1$.

For convenience, we denote

$$\Delta_i := \left\{\sum_{j=1}^{N} \max_{n\in\mathbb{Z}^{[0,\theta]}} |w_{ij}(n)| + \max_{n\in\mathbb{Z}^{[0,\theta]}} |\lambda_i(n)|\right\}, \quad \lambda_i(n) := a_i(n) - w_{ii}(n)a_{0i}(n).$$

For any initial condition $\psi \in \Omega$ which defined by

$$\Omega := \left\{\psi \in S\big(\mathbb{Z}^{[-\upsilon,0]}, \mathbb{R}^N\big) \Big| -\frac{\Delta_i}{1-K} \leqslant \psi^i(s) \leqslant \frac{\Delta_i}{1-K}, \quad i \in \mathscr{N} \text{ and } s \in \mathbb{Z}^{[-\upsilon,0]}\right\},$$

it is easy for us to get $y_n(\cdot;\psi) \in \Omega$ for all $n \in \mathbb{Z}^{[0]}$.

Lemma 2.1. *Assume that* (H_1) *and*

- $(H_1^{\mathscr{A}})$ $0 < \epsilon \max\limits_{n\in\mathbb{Z}^{[0,\theta]}}\{1-k(n)\} \Big/ \min\limits_{n\in\mathbb{Z}^{[0,\theta]}}\{w_{ii}(n)\} < \dfrac{1}{4}, \quad i \in \mathscr{N}$

hold. Then for each $i \in \mathscr{N}$, *there exist only two points* ξ_{i1} and ξ_{i2} *with* $\xi_{i1} < 0 < \xi_{i2}$ *such that* $\dot{H}_i(\xi_{iq}) = 0$ $(q = 1,2)$ and $\dot{H}_i(z) \cdot sgn\left\{\frac{z-\xi_{i1}}{z-\xi_{i2}}\right\} < 0$, *where*

$$H_i(z) := -\max_{n\in\mathbb{Z}^{[0,\theta]}}\{1-k(n)\}z + \min_{n\in\mathbb{Z}^{[0,\theta]}}\{w_{ii}(n)\}\big/\{1+e^{-z/\epsilon}\}.$$