Commun Nonlinear Sci Numer Simulat 15 (2010) 1063–1070

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Impulsive synchronization of general continuous and discrete-time complex dynamical networks

Qunjiao Zhang [a,*], Jun-an Lu [b], Junchan Zhao [b]

[a] College of Science, Wuhan University of Science and Engineering, Wuhan, Hubei 430073, PR China
[b] School of Mathematics and Statistics, Wuhan University, Wuhan, Hubei 430072, PR China

ARTICLE INFO

Article history:
Received 20 January 2009
Received in revised form 8 May 2009
Accepted 9 May 2009
Available online 27 May 2009

PACS:
05.45.Xt
05.45.-a

Keywords:
Impulsive synchronization
Dynamical networks
Continuous
Discrete-time
Chen system
Hénon map

ABSTRACT

This paper mainly investigates the impulsive synchronization of a general complex continuous and discrete-time dynamical network. Firstly, for the continuous complex networks, we give a sufficient condition to guarantee its synchronization. When the sufficient condition is not satisfied, the impulsive controllers are utilized, and some novel criteria are derived to guarantee the network synchronization in this case. What is more significant is that the similar work is extended to the discrete-time networks model. Finally, the results are, respectively, illustrated by a continuous network composed with the chaotic Chen oscillators and a discrete-time network consisting of Hénon map. All numerical simulations verify the effectiveness of the theoretical analysis.

1. Introduction

Complex dynamical networks are attracting more and more attention due to their ubiquity in the nature world. In our daily life, a great number of natural complex networks such as cooperate networks, social networks, neural networks, WWW, food webs, electrical power grids and so on are widely studied by the researchers. Particularly, one of the most interesting and significant phenomena in complex dynamical networks is the synchronization of all dynamical nodes [1–3], it has been demonstrated that many real-world problems have close relationships with network synchronization, such as the lighting of fireflies, and the spread of an epidemic or computer virus. Over the past years, the synchronization of networks had been deeply researched by many scientists from various fields, for instance, sociology, biology, mathematics and physics [4–12].

Generally speaking, there are three primary approaches to be used to study the synchronization of networks, that is, Lyapunov–Krasovskii function direct method, the Master Stability Function (MSF) method, Connection Graph Stability (CGS) method. Besides, there are several control schemes have been introduced to realize the network synchronization, for example, the adaptive synchronization [13], pinning control [14–17] and robust impulsive synchronization [18], etc.

In practical implement, among these approaches, it has been proved that the impulsive synchronization method is effective and relatively easily realized [19,20]. It allows synchronization of a complex network only by small impulses being sent

* Corresponding author.
E-mail address: qunjiao99@163.com (Q. Zhang).

1007-5704/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2009.05.048

to the receive systems at the discrete impulsive instances, which can reduce the information redundancy in the transmitted signal and increase robustness against the disturbances. In this sense, impulsive synchronization scheme has been applied to numerous chaos-based communication systems for cryptographics secure purpose and detailed experiments had been carried out [21–23].

In this paper, we consider the impulsive synchronization problem of continuous and discrete-time dynamical networks with general topology. Firstly, for the continuous complex networks, we obtain a sufficient condition to guarantee its synchronization. When the sufficient condition is not satisfied, the impulsive control method is utilized, and some novel criteria are derived to realize the network synchronization in this case. What is more significant is that the similar work is extended to the discrete-time networks model. Finally, the results are, respectively, illustrated by a continuous network composed with the chaotic Chen oscillators and a discrete-time network consisting of Hénon map. All involved numerical simulations verify the correctness of the theoretical analysis.

The notations used throughout this paper are fairly standard. The superscript "T" stands for matrix transposition; R^n denotes the n-dimensional Euclidean space; I is the identity matrix. Furthermore, for a given matrix $A = (a_{ij}) \in R^{n\times n}$, we define $\|A\| = \sqrt{\lambda_{max}(A^TA)}$ and the characteristic quality $\mu(A) = \lambda_{max}\frac{1}{2}(A^T + A)$, where $\lambda(\cdot)$ is the eigenvalue of matrix $\cdot$.

2. Impulsive synchronization of continuous complex dynamical networks

Consider a continuous complex network consisting of N nonlinearly coupled identical nodes and each node is an m-dimensional autonomous dynamical system. The equations of the entire network are

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} c_{ij}h(x_j(t)), \quad i = 1,2,\ldots,N \tag{1}$$

where $f : R^n \to R^n$ is a continuously differentiable function, $x_i(t) = (x_{i1}(t), x_{i2}(t),\ldots,x_{im}(t))^T \in R^m$ are state variables of node i, the constant c is the coupling strength, the map $h(\cdot)$ is the inner connecting function in each node and the matrix $C = (c_{ij}) \in R^{N\times N}$ is outer coupling matrix of the networks, which is defined as follows: $\sum_{j=1}^{N} c_{ij} = 0$ and $c_{ij} = c_{ji} = 1 (i\neq j)$ if there is a connection between node i and j $(j\neq i)$, otherwise, $c_{ij} = c_{ji} = 0 (i\neq j)$.

Definition 1. The nonlinear coupled network (1) is said to achieve asymptotical synchronization if

$$x_i(t) \to \bar{x} \ \text{ as } t \to +\infty, \quad i = 1,2,\ldots,N,$$

where $\bar{x}$ is an equilibrium point, satisfying $f(\bar{x}) = 0$.

Lemma 1. *If the following linear systems are asymptotically stable about their zero solutions:*

$$\dot{w}(t) = [Df(\bar{x}) + c\lambda_k Dh(\bar{x})]w(t), \quad k = 1,2,\ldots,N, \tag{2}$$

where $Df(\bar{x}), Dh(\bar{x}) \in R^{m\times m}$ are, respectively, the Jacobian of $f(\cdot)$ and $h(\cdot)$ at $\bar{x}$. Then the continuous complex dynamical network (1) is asymptotically synchronous.

Proof. Let $e_i(t) = x_i(t) - \bar{x}$ $(i = 1,2,\cdots,N)$. Notice that $f(\bar{x}) = 0$ and $\sum_{j=1}^{N} c_{ij} = 0$, one can yield the variation equations as below:

$$\begin{aligned}\dot{e}_i(t) = \dot{x}_i(t) &= f(x_i(t)) - f(\bar{x}) + c\sum_{j=1}^{N} c_{ij}h(x_j(t)) - c\sum_{j=1}^{N} c_{ij}h(\bar{x}) = Df(\bar{x})e_i(t) + c\sum_{j=1}^{N} c_{ij}Dh(\bar{x})e_j(t) \\ &= Df(\bar{x})e_i(t) + cDh(\bar{x})(e_1(t),\ldots,e_N(t))(c_{i1}(t),\ldots,c_{iN}(t))^T, \quad i = 1,2,\ldots,N.\end{aligned} \tag{3}$$

Let $E^T = [e_1(t),\ldots,e_N(t)] \in R^{m\times N}$, we rewrite Eq. (3) as

$$\dot{E} = E(Df(\bar{x}))^T + cCE(Dh(\bar{x}))^T. \tag{4}$$

From the assumption character of matrix C, we denote its eigenvalues as sequences

$$\lambda_N \leqslant \ldots \leqslant \lambda_2 < \lambda_1 = 0.$$

According to the theory of Jordan canonical forms, there exists a generalized eigenvectors matrix

$$\Phi = [\phi_1, \phi_2, \ldots, \phi_N] \in R^{N\times N}$$

such that

$$C = \Phi\Lambda\Phi^T,$$

where $\Phi^T\Phi = I$ and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$.

Introducing a nonsingular linear transformation $E = \Phi\eta$, where $\eta^T = [\eta_1(t),\ldots,\eta_N(t)] \in R^{m\times N}$. Along with (4), we have

$$\dot{\eta} = \eta(Df(\bar{x}))^T + c\Lambda\eta(Dh(\bar{x}))^T, \tag{5}$$