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Short communication

Slip MHD viscous flow over a stretching sheet – An exact solution

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ABSTRACT

In this paper, the magnetohydrodynamic (MHD) flow under slip condition over a permeable stretching surface is solved analytically. The solution is given in a closed form equation and is an exact solution of the full governing Navier–Stokes equations. The effects of the slip, the magnetic, and the mass transfer parameters are discussed. Results show that there is only one physical solution for any combination of the slip, the magnetic, and the mass transfer parameters. The velocity and shear stress profiles are greatly influenced by these parameters.

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1. Introduction

The flow induced by a moving boundary is important in the extrusion processes [1–3]. Sakiadis studied the boundary layer flow on a continuously stretching surface with a constant speed [4,5]. However, according to Wang [6], Sakiadis' solution was not an exact solution of the whole Navier–Stokes (NS) equations. Crane [7] presented an exact solution of the two-dimensional NS equations for a stretching sheet problem. The effect of mass transfer on the Crane flow was investigated by Gupta and Gupta [8]. The stretching boundary problem was extended by Wang [9] to a three-dimensional setting. Exact similarity solutions of the NS equations were obtained. The result of Wang for a stretching disk was extended to rotating flow over a stretchable disk [10]. The flow between two stretchable disks was also studied [11]. All these solutions are fortunately exact solutions of the NS equations. The magnetohydrodynamic (MHD) flow over a stretching sheet was studied in the literature [12–15] for both impermeable surface and permeable surface. In the recent years, micro-scale fluid dynamics in the Micro-Electro-Mechanical Systems (MEMS) received much attention in research. Because of the micro-scale dimensions, the fluid flow behavior belongs to the slip flow regime and greatly differs from the traditional flow [16]. For the flow in the slip regime, the fluid motion still obeys the Navier–Stokes equations, but with slip velocity or temperature boundary conditions. In addition, partial velocity slips over a moving surface occur for fluids with particulate such as emulsions, suspensions,

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foams, and polymer solutions [17]. The slip flows under different flow configurations have been studied in the literature [18–23]. With a slip at the wall boundary, the flow behavior and the shear stress in the fluid are greatly different from the no-slip conditions. The solutions provided in these publications are all exact solutions of the NS equations. Among these papers, the slip flow over an impermeable stretching surface was studied by Andersson [18] and Wang [19]. In a recent paper, the mass transfer effects on the slip flow over a stretching surface were reported [23]. However, there was no study on the slip MHD flow over a permeable stretching surface in the literature. Therefore, the objective of the current paper is to study the slip MHD flow over a permeable stretching surface. Exact solutions of the governing NS equations are presented and discussed.

2. Mathematical formulation and discussion

Consider a steady, two-dimensional laminar flow over a continuously stretching sheet in an electrically conducting quiescent fluid. The sheet stretching velocity is $U_w = U_0(x)$ and the wall mass transfer velocity is $v_w = v_w(x)$, which will be determined later. The x-axis runs along the shrinking surface in the direction opposite to the sheet motion and the y-axis is perpendicular to it. The governing NS equations of this problem read [12–14,18,19]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial u} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial u} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

with the boundary conditions (BCs)

$$u(x,0) = U_0 x + L\frac{\partial u}{\partial y}, \quad v(x,0) = v_w(x), \quad \text{and} \quad u(x,\infty) = 0 \tag{4a--4c}$$

where u and v are the velocity components in the x and y directions, respectively, ν is the kinematic viscosity, p is the fluid pressure, ρ is the fluid density, and σ is the electrical conductivity of the fluid, and L is a proportional constant of the velocity slip. The magnetic field with strength B is applied in the vertical direction and the induced magnetic field is neglected. This group of NS equations is valid for small magnetic field strength. The stream function and the similarity variable can be posited in the following form,

$$\psi(x,y) = f(\eta)x\sqrt{\nu U_0} \quad \text{and} \quad \eta = y\sqrt{\frac{U_0}{\nu}} \tag{5a--5b}$$

With these definitions, the velocities are expressed as $u = U_0 x f'(\eta)$ and $v = -\sqrt{U_0 \nu} f(\eta)$. The wall mass transfer velocity becomes $v_w(x) = -\sqrt{U_0 \nu} f(0)$. The similarity equation is obtained as follows

$$f''' + ff'' - f'^2 - M^2 f' = 0 \tag{6}$$

with the BCs as follows

$$f(0) = s, \quad f'(0) = 1 + \gamma f''(0), \quad \text{and} \quad f'(\infty) = 0 \tag{7a--7c}$$

where s is the wall mass transfer parameter showing the strength of the mass transfer at the sheet, M is the magnetic parameter with $M^2 - \frac{\sigma B^2}{\rho U_0}$, and γ is the velocity slip parameter with $\gamma = L\sqrt{\frac{U_0}{\nu}}$. The pressure term can be obtained from Eq. (3) as $\frac{p}{\rho} = \nu\frac{\partial v}{\partial y} - \frac{v^2}{2} + constant$. There is an analytical solution for $\gamma = 0$ [14] as

$$f(\eta) = s + \frac{1}{\frac{s+\sqrt{s^2+(4+4M^2)}}{2}} - \frac{1}{\frac{s+\sqrt{s^2+(4+4M^2)}}{2}} e^{-\frac{s+\sqrt{s^2+(4+4M^2)}}{2}\eta} \tag{8}$$

In this paper, we will show a closed form exaction solution of Eq. (6) together with the BCs. Assuming the solution has a format as $f(\eta) = a + be^{-\beta\eta}$. Substituting this relation to Eq. (6) yields

$$b = -\frac{1}{\beta + \gamma\beta^2}, \tag{9}$$

$$a = s + \frac{1}{\beta + \gamma\beta^2} \tag{10}$$

and β is the root of the following third order algebraic equation

$$\gamma\beta^3 + (1 - s\gamma)\beta^2 - (s + \gamma M^2)\beta - 1 - M^2 = 0 \tag{11}$$

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