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Transmission of solitary pulse in dissipative nonlinear transmission lines

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ABSTRACT

A class of dissipative complex Ginzburg–Landau (DCGL) equations that govern the wave propagation in dissipative nonlinear transmission lines is solved exactly by means of the Hirota bilinear method. Two-soliton solutions of the DCGL equations, from which the one-soliton solutions are deduced, are obtained in analytical form. The modified Hirota method imposes some restrictions on the coefficients equations. Namely, the second-order dispersion must be real. The physical requirement of the solutions imposes complementary conditions on the combination of the dispersion and nonlinear gain/loss terms of the equation, as well as on the coefficient of the Kerr nonlinearity. The analytical solutions for one-solitary pulses are tested in direct simulations.

1. Introduction

Since the 1970s, various investigators have discovered the existence of solitons in nonlinear transmission lines (NLTLs), through both mathematical models and physical experiments (see for example Refs. [1–5]). Scott's classical monograph [6] was among the first to treat the physics of transmission lines. Scott showed that the Korteweg–deVries (KdV) equation describes weakly nonlinear waves in a nonlinear LC transmission line containing a finite number of cells which consist of two elements: a linear inductor in the series branch and a nonlinear capacitor in the shunt branch. If the nonlinearity is moved from the capacitor parallel to the shunt branch of the line to a capacitor parallel to the series branch, the nonlinear Schrödinger (NLS) equation is obtained instead [7].

Some years ago, the nonlinear propagation of signals in electrical transmission lines has been investigated, theoretically and experimentally [5,8,9]. It has been shown that the system of equations governing the physics of the considered network can be reduced either to single or coupled NLS equations or to the Ginzburg–Landau (GL) equations. The single and the coupled NLS equations admit the formation of envelope solitons, which have been observed experimentally [8,9].

More recently, Pelap et al. [10] presented a model for wave propagation on a discrete dissipative electrical transmission line of Fig. 1 based on the complex Ginzburg–Landau (CGL) equation

$$i\frac{\partial A}{\partial t}+P\frac{\partial^2 A}{\partial x^2}+Q\mid A|^2A=i\gamma A, \tag{1}$$

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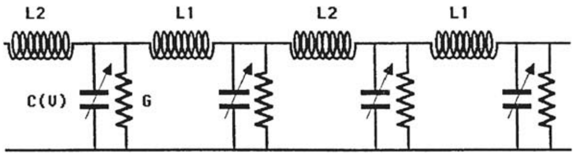

Fig. 1. Representation of a discrete nonlinear dissipative bi-inductance transmission line.

derived in the small amplitude and long wavelength limit using the standard reductive perturbation technique and complex expansion [11] of the governing nonlinear equations. Here A is a complex amplitude, $P = P_r + iP_i$ and $Q = Q_r + iQ_i$ are two complex constants, and γ is a positive constant. Basically, the evolution of the complex wave envelope A is controlled by the competition of the dispersion ($\sim P$), nonlinearity ($\sim Q$), and linear gain γ. Physically, P_r measures the wave dispersion, P_i measures the relative growth rate of disturbances whose spectra are concentrated near the fundamental wavenumber k, Q_r determines how the wave frequency is amplitude modulated, Q_i measures the saturation of the unstable wave, and the positive constant γ is the linear gain. The modulational instability of the Stokes waves $A(x,t) = A_0 \exp[ik_0x - i(P_rk_0^2 - Q_r \mid A_0|^2)t]$, where $Q_i \mid A_0|^2 = \gamma + P_ik_0^2$, is considered in Ref. [10] and the modulational instability criterion $P_rQ_r + P_iQ_i > 0$ has been found.

In this work, we study the transmission of solitary pulses, governed by the GL Eq. (1), propagating in the network of Fig. 1. The paper is structured as follows. In Section 2, we show how the modified Hirota method is applied to construct the exact solitary pulse solutions of the DCGL equation. In Section 3, we present some numerical results, and the paper is concluded in Section 4.

2. Derivation of explicit solitary pulse solutions

In order to prove that Eq. (1) can support envelope solitons, we use the Hirota bilinear technique [12]. Thus, we follow the definition of Nozaki and Bekki [13,14] and introduce the modified Hirota derivative

$$D^m_{\alpha,t}D^n_{\alpha,x}F.G = \left(\frac{\partial}{\partial t'} - \alpha\frac{\partial}{\partial t}\right)^m \left(\frac{\partial}{\partial x'} - \alpha\frac{\partial}{\partial x}\right)^n F(x,t)G(x',t')\bigg|_{x'=x,t'=t}. \quad (2)$$

We first note that Eq. (1), under the transformation $A(x,t) = \psi(x,t)\exp(\gamma t)$, takes the form

$$i\psi_t + P\psi_{xx} + Q\exp(2\gamma t) \mid \psi|^2\psi = 0. \quad (3)$$

The two-soliton solution of Eq. (3) is given by

$$\psi(x,t) = u_1(x,t) + u_2(x,t), \quad (4)$$

where $u_1(x,t)u_2(x,t) = 0$ corresponds to the single soliton. Inserting Eq. (4) into Eq. (3), we obtain the system

$$\begin{aligned} &i\frac{\partial u_1}{\partial t} + P\frac{\partial^2 u_1}{\partial x^2} + Q\exp(2\gamma t) \mid u_1|^2u_1 + Q\exp(2\gamma t)(2 \mid u_1|^2u_2 + u_1^2u_2^*) = 0,\\ &i\frac{\partial u_2}{\partial t} + P\frac{\partial^2 u_2}{\partial x^2} + Q\exp(2\gamma t) \mid u_2|^2u_2 + Q\exp(2\gamma t)(2 \mid u_2|^2u_1 + u_2^2u_1^*) = 0, \end{aligned} \quad (5)$$

where "*" stands for the complex conjugate. To obtain exact solutions of system (5) we adopt the modified Hirota ansatz

$$u_1(x,t) = G(x,t)F^{-\alpha}(x,t), \quad u_2(x,t) = H(x,t)F^{-\alpha}(x,t), \quad (6)$$

where G and H are two complex functions, F is a real function, and α is, in general, complex. Due to the presence of power $-\alpha$, transformation (6) is different from that used in the case of the conventional nonlinear Schrödinger system. This difference is, as a matter of fact, the main motivation for introducing these modified Hirota derivatives. Expression (6) is deduced from the truncation of the Puiseux expansions at the lowest level [15]. Using Eqs. (6), (5) is rewritten as a pair of bilinear equations in terms of the modified Hirota derivative (2),

$$[iD_{\alpha,t} + PD^2_{\alpha,x} + iP\beta + 2ix\beta PD_{\alpha,x}]F.G = 0, \quad [iD_{\alpha,t} + PD^2_{\alpha,x} + iP\beta + 2ix\beta PD_{\alpha,x}]F.H = 0, \quad (7)$$

$$D^2_xF.F = \frac{2Q\exp(2\gamma t)(2HG^* + GH^* + GG^*)}{P\alpha(1+\alpha)F^{2\mathrm{Re}}\alpha - 2}, \quad D^2_xF.F = \frac{2Q\exp(2\gamma t)(2GH^* + HG^* + HH^*)}{P\alpha(1+\alpha)F^{2\mathrm{Re}}\alpha - 2}, \quad (8)$$

for $GH \neq 0$. So the left-hand sides of Eq. (8) become equal. Hence the right-hand sides of Eq. (8) should also be equal which is true only under the bilinear condition

$$(H+G)(G^* - H^*) = 0, \quad (9)$$

that is, $H = \varepsilon G$ with $\varepsilon = \pm 1$. In what follows, we consider that $\varepsilon \in \{-1, 0, 1\}$, $\varepsilon = 0$ corresponding to the single soliton propagating in the network. We then obtain