Commun Nonlinear Sci Numer Simulat 14 (2009) 3945–3952

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Synchronization of delayed fuzzy cellular neural networks with impulsive effects

Wei Ding *

Department of Mathematics, Shanghai Normal University, Shanghai 200234, China

ARTICLE INFO

Article history:
Received 24 October 2008
Received in revised form 17 February 2009
Accepted 17 February 2009
Available online 28 February 2009

PACS:
92B20
92C20
94B50
94D05

Keywords:
Fuzzy cellular neural networks
Synchronization
Delay
Impulses

ABSTRACT

This letter studies synchronization of delayed fuzzy cellular neural networks with impulses and all the parameters unknown. To avoid the difficulties which may be brought by the impulses, a non-impulsive system is used to replaced the system with impulses. Then by the well known Lyapunov–Lasall principle, some new stability criteria are obtained. An example and its simulation were given to illustrate the simpleness and effectiveness of our main results.

1. Introduction

Synchronization have attracted much attention for the important applications in varies areas after it is proposed by Pecora and Carrol [1,2]. Considering the effect of the past history which plays important role in real life, many works deal with synchronization phenomena in delayed neural networks [3–8], etc. For instance, based on Lynapunov functional method and Hermitian matrices theory, Chen [3] proposed global synchronization for a class of delayed neural networks. Ref. [4] studied synchronization of coupled chaotic systems to delayed system in the presence of unknown parameters. Cao [5] investigated the adaptive synchronization of neural networks with and without time-varying delays.

It is well known, many evolution processes do exhibit impulsive effects. These days, impulsive control synchronization has been developed [6–10]. In 2007, Yang and Cao [6] considered a class of delayed neural networks with the form,

$$\begin{aligned}
&\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(y_j(t-\tau_{ij})) + I_i, \quad t \neq t_k, t \geqslant 0, \\
&\triangle y_i(t_k) = -\gamma_{ik} y(t_k), \quad k = 1, 2, \cdots, \\
&y_i(t) = \phi_i(t), \quad t \in [-\tau, 0],
\end{aligned} \tag{1.1}$$

* Tel.: +8621 64323266.
E-mail address: dingwei@shnu.edu.cn

1007-5704/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2009.02.013

The main result of [6] was obtained by constructing properly $V-$ *functional*. That is, system (1.1) is exponential lag synchronization with the following slave system,

$$\begin{aligned}\dot{x}_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + I_i \\ &\quad + d_i(y_i(t-\sigma) - x_i(t)), \quad t \neq t_k + \sigma, \quad t \geqslant \sigma, \\ \triangle x_i(s_k) &= -\gamma_{ik} x(s_k),\ s_k = t_k + \sigma, \quad k = 1, 2, \cdots \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau + \sigma, \sigma].\end{aligned}$$

where d_i denotes the controller gain. In the following year, Zhu et al. [7] studied impulsive exponential synchronization of a class of chaotic systems by using an impulsive inequality and the property of P-cone.

However, even smallest perturbations may bring about the failure of synchronization scheme [10]. Yang et al. put forward the fuzzy theorem [11,12] in 1996. They integrated fuzzy logic into the structure of a traditional cellular neural networks, and maintained local connectedness among cells. The added fuzzy logic can be seen as the perturbations. Kuan [13] and Ding [14] studied exponential stability, periodic solutions, and synchronization, respectively. The system they concerned is the following,

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} v_j + L_i + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau_{ij})) + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau_{ij})) + \bigvee_{j=1}^{n} S_{ij} v_j, \ \ i = 1, 2, \cdots, n.$$

Inspired by the aforementioned discussion, in this paper, we investigate a class of delayed fuzzy cellular neural network with impulses. Moreover, since the coefficients of some system cannot be exactly known, it is necessary to consider the uncertainty parameters. Compare with the recent literatures, the main difficulties in this letter lie in two aspects. First, the difficulties in mathematical analysis which impulses may bring. Second, the difficulties in deriving the asymptotical property of unknown parameters.

The paper is organized as follows. In Section 2, we first prove the system with impulses is equivalence to the system without impulses. Then, some preliminaries which are necessary to proof of the main results are established. Section 3 derives some sufficient conditions to ensure some synchronization criteria. In Section 4, an example and its simulations are given to illustrate the simpleness and effectiveness of our main results. Conclusions are drawn in the last section.

2. Preliminaries

In this paper, we study the following system,

$$\begin{aligned}\dot{x}_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} v_j + L_i \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau_{ij})) + \bigwedge_{j=1}^{n} T_{ij} v_j \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau_{ij})) + \bigvee_{j=1}^{n} S_{ij} v_j, \quad t \geqslant 0, \quad t \neq t_k \\ \triangle x_i(t_k) &= I_{ik} x(t_k), \quad k = 1, 2, \cdots, m. \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau, 0],\end{aligned} \tag{1}$$

where $\alpha_{ij}, \beta_{ij}, T_{ij}, S_{ij}$ are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template, fuzzy feed-forward MAX template, respectively. a_{ij}, b_{ij} are elements of feedback and feed-forward template. $\bigwedge, \bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. x_i, v_i and u_i denote state, input and bias of the ith neuron, respectively. τ_{ij} are the transmission delay, and f_j are the activation functions. $I_{ik}, k = 1, 2, \cdots,\ i = 1, 2, \cdots, n$ denote impulsive gain set. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ is the impulses at moments t_k, $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$, and functions $\phi_i(.)$ are bounded and continuous on $[-\tau, 0], \tau \geqslant 0$.

As usual in the theory of impulsive differential equations, at the points of discontinuity $t_k, k = 1, 2, \cdots, m$, we assume that $x_i(t_k) \equiv x_i(t_k^-)$ and $x_i'(t_k) \equiv x_i'(t_k^-)$.

Inspired by Ref. [15], we construct a equivalent theorem between (1) and (2). Then we establish some lemmas which are necessary in the proof of the main results.

In this paper, we assume the following conditions are hold.

(A_1) The impulses $I_{ik} \neq -1, \quad k = 1, 2, \cdots, \ i = 1, 2, \cdots, n$

(A_2) There exist constants $k_i \geqslant 0$, such that for any $x_1, x_2 \in R$, there has

$$|f_i(x_1) - f_i(x_2)| \leqslant k_i |x_1 - x_2|, \quad i = 1, 2, \cdots, n.$$