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Mathematical analysis of a tuberculosis model with differential infectivity ☆

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ABSTRACT

This paper deals with the global properties of a tuberculosis model with mass action incidence and two differential infectivity. The direct Lyapunov method enables us to prove that the considered model is globally stable: There is always a globally asymptotically stable equilibrium state. Depending on the value of the basic reproduction number R_0, this state can be either endemic ($R_0 > 1$), or infection-free ($R_0 \leqslant 1$). Numerical results are provided to illustrate analytical results.

1. Introduction

Tuberculosis (TB) is a disease caused by infection with Mycobacterium tuberculosis, which most frequently affects the lungs (pulmonary TB). It is one of the most common infectious diseases with about two billion people (one-third of the world's population) currently infected. About nine million new cases of active disease develop each year, resulting in two million deaths, mostly in developing countries. Despite intensive control efforts, recent data show that global incidence is increasing, largely due to an association with human immunodeficiency virus (HIV) [1].

Mathematical models have played a key role in the formulation of TB control strategies and the establishment of interim goals for intervention programs [2–13]. Most of these models are of the SEIR class in which the host population is categorized by infection status as susceptible, exposed (infected but not yet infectious), infectious and recovered. One of the principle attributes of these models is that the force of infection (the rate at which susceptible leave the susceptible class and move into an infected category, i.e., become infected) is a function of the number of infectious hosts in the population at any time t and is thus a nonlinear term. Other transitions, such as the recovery of infectious and death, are modelled as linear terms with constant coefficients. However, the enormous public health burden inflicted by tuberculosis necessitates

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the use of mathematical modelling to gain insights into it transmission dynamics and to determine effective control strategies.

Motivated by TB models in the literature [2–13], this paper investigates the global properties of a deterministic model for tuberculosis transmission dynamics with two differential infectivity that incorporate constant recruitment, slow and fast progression, effective chemoprophylaxis (given to latently infected individuals) and therapeutic treatments (given to infectious). To the best of authors knowledge, the global analysis of tuberculosis models with differential infectivity is not well discussed in the literature. With respect to TB models in the literature [2–13], we introduce a new epidemiological class that we call the class of loss of sight. By loss of sight, we mean infectious that begun their effective therapy in the hospital and never return to the hospital for the spuctrum examinations for many reasons such as long duration of treatment regimen, poverty, mentality, etc... In this case, the health personal don't know if they are died, recovered or not. One reason to introduce this new epidemiological class is that loss of slight usually occur in Sub-Saharan Africa. For example, according to the National Program of Fight against Tuberculosis of Cameroon, about 10% of infectious that begun their therapy treatment become loss of slight. Therefore, this fact cannot be neglected in the modelling of TB.

We introduce a direct transfer from the class of susceptible individual toward the compartment of infectious. The reason is the co-infection with HIV infection. We also incorporate a transfer from the infectious to the exposed to take into account that some infectious apparently recover, but actually harbor TB bacteria. The global dynamics of the model is resolved through the use of Lyapunov functions. The Lyapunov functions used in this paper to demonstrate the global stability of the endemic equilibrium is the same from as those used recently in Refs. [16–28] to determine the global dynamics of SEIR, SEIS, and SIR models.

The paper is structured as follows. Section 2 presents a new tuberculosis model with two differential infectivity. In Section 3, we present the global properties of the proposed model. The basic reproduction ratio of infection is obtained and used to determine conditions for the existence and uniqueness of associated equilibrium points. Lyapunov functions are constructed to establish the global asymptotic stability of the disease-free and endemic equilibria. Numerical simulation are provided to validate analytical results. We also presented a numerical comparative between our model and the SEIR model proposed by Porco and Blower [7]. Finally, section 4 contains conclusions.

2. Model construction

In this section, we formulate a model for the spread of tuberculosis in a human population. Fig. 1 shows the model diagram. Based on epidemiological status, the population is divided into five classes: susceptible (S), infected (E), infectious (I) and loss of sight (L). All recruitment is into the susceptible class, and occurs at a constant rate Λ. The rate constant for non-disease related death is μ, thus $1/\mu$ is the average lifetime. Infectious and loss of sight have addition death rates due to the disease with rates constant d_1 and d_2, respectively. Since we don't know if loss of slight are recovered, died or are still infectious, we assume that a fraction δ of them is still infectious and can transmit disease to susceptible. Transmission of M. tuberculosis occurs following adequate contacts between a susceptible and infectious or loss of sight that continue to have disease. We assume that infected individuals are not infectious, and thus not capable of transmitting bacteria. We use the standard mass balance incidence expressions βSI and $\beta\delta SL$ to indicate successful transmission of M. tuberculosis due to nonlinear contacts dynamics in the population by infectious and loss of slight, respectively. A fraction p of the newly infected individuals are assumed to undergo fast progression directly to the infectious class, while the remainder are latently infected and enter the latent class. Once latently infected with M. Tuberculosis, an individual will remain so for life unless reactivation occurs. To account for treatment, we define r_1E as the fraction of infected individuals receiving effective chemoprophy-

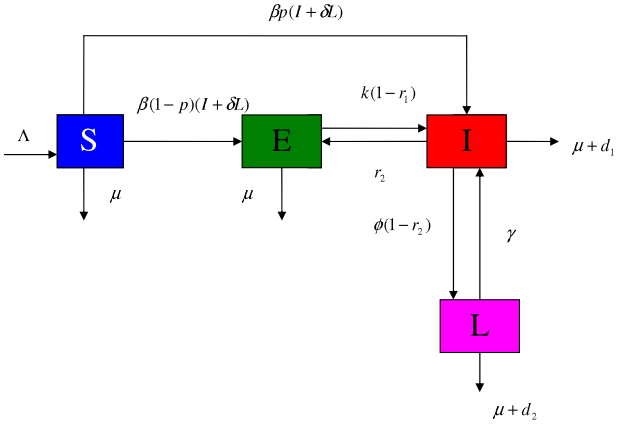

Fig. 1. Transfer diagram for the dynamics transmission of tuberculosis with two differential infectivity.