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Pulsatile flow of an unsteady dusty fluid through rectangular channel

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ABSTRACT

The geometry of laminar flow of an unsteady viscous fluid with uniform distribution of dust particles through a rectangular channel under the influence of pulsatile pressure gradient has been considered. Initially the fluid and dust particles are at rest. The analytical expressions for velocities of fluid and dust particles is obtained by solving the partial differential equations using variable separable method and Laplace transform technique. The skin friction at the boundary plates are also calculated. Finally the changes in the velocity profiles with s and n are shown graphically.

1. Introduction

The fluid flow embedded with dust particles is encountered in a wide variety of engineering problems concerned with atmospheric fallout, dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, batch settling, rainerosion, guided missiles and paint spraying etc.

Saffman [14] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Liu [11] has studied the flow induced by an oscillating infinite plat plate in a dusty gas. Michael and Miller [12] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in the semi-infinite space above a rigid plane boundary. Ghosh et al. [15] have obtained the analytical solutions for the dusty visco-elastic fluid between two infinite parallel plates under the influence of time dependent pressure gradient, using appropriate boundary conditions. Authors like Amos [1] and Datta [6] have studied the fluid flow due to pulsatile pressure gradient.

Some researchers like Kanwal [10], Truesdell [16], Indrasena [9], Purushotham and Indrasena [13], Bagewadi, Shantharajappa and Gireesha [2–4] have applied differential geometry techniques. Further, the authors [3,4] have studied two-dimensional dusty fluid flow in Frenet frame field system. Recently the authors [7,8] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential. In the present paper laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through a rectangular channel under the influence of pulsatile pressure gradient in Frenet frame field system is considered. Further by considering the fluid and dust

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particles to be at rest initially, the analytical expressions are obtained for velocities of fluid and dust particles also the skin friction at the boundary is calculated. The graphical representation of the velocity profiles versus s are given.

2. Equations of motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [14]:

For fluid phase:

$$\nabla \cdot \vec{u} = 0 \quad \text{(Continuity)} \tag{2.1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\rho^{-1}\nabla p + \nu\nabla^2\vec{u} + \frac{kN}{\rho}(\vec{v} - \vec{u}) \quad \text{(Linear momentum)} \tag{2.2}$$

For dust phase:

$$\nabla \cdot \vec{v} = 0 \quad \text{(Continuity)} \tag{2.3}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{k}{m}(\vec{u} - \vec{v}) \quad \text{(Linear momentum)} \tag{2.4}$$

We have following nomenclature:

$\vec{u}$ is the velocity of the fluid phase, $\vec{v}$ is the velocity of dust phase, ρ is the density of the gas, p is the pressure of the fluid, N is the number of density of dust particles, ν is the kinematic viscosity, $k = 6\pi a\mu$ is the Stoke's resistance (drag coefficient), a is the spherical radius of dust particle, m is the mass of the dust particle, μ is the the coefficient of viscosity of fluid particles, and t is the time.

Let $\vec{s}, \vec{n}, \vec{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal, respectively, to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines, respectively, as shown in Fig. 1.

Geometrical relations are given by Frenet formulae [5]

$$\begin{aligned}
&\text{(i)} \quad \frac{\partial \vec{s}}{\partial s} = k_s \vec{n}, \quad \frac{\partial \vec{n}}{\partial s} = \tau_s \vec{b} - k_s \vec{s}, \quad \frac{\partial \vec{b}}{\partial s} = -\tau_s \vec{n} \\
&\text{(ii)} \quad \frac{\partial \vec{n}}{\partial n} = k'_n \vec{s}, \quad \frac{\partial \vec{b}}{\partial n} = -\sigma'_n \vec{s}, \quad \frac{\partial \vec{s}}{\partial n} = \sigma'_n \vec{b} - k'_n \vec{n} \\
&\text{(iii)} \quad \frac{\partial \vec{b}}{\partial b} = k''_b \vec{s}, \quad \frac{\partial \vec{n}}{\partial b} = -\sigma''_b \vec{s}, \quad \frac{\partial \vec{s}}{\partial b} = \sigma''_b \vec{n} - k''_b \vec{b} \\
&\text{(iv)} \quad \nabla \cdot \vec{s} = \theta_{ns} + \theta_{bs}; \quad \nabla \cdot \vec{n} = \theta_{bn} - k_s; \quad \nabla \cdot \vec{b} = \theta_{nb}
\end{aligned} \tag{2.5}$$

where $\partial/\partial s, \partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsions of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal, respectively.

3. Formulation and solution of the problem

Consider a flow of viscous incompressible, dusty fluid through a rectangular channel. The flow is due to the influence of pulsatile pressure gradient varying with time. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust par-

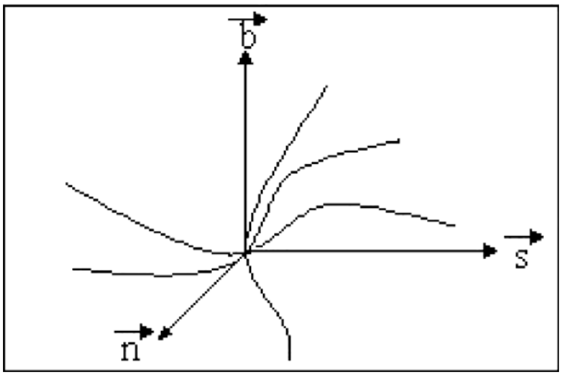

Fig. 1. Frenet frame field system. *Note*: Graphs are drawn for the values of $h_1 = -1, h_2 = 1, r_1 = 1, r_2 = 3, \tau = 0.5, \beta = 1, \alpha = 1, C_r = 1, Q = 1, l = 1, C = 1$.