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Finite-time chaos synchronization of unified chaotic system with uncertain parameters ☆

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A B S T R A C T

This paper deals with the finite-time chaos synchronization of the unified chaotic system with uncertain parameters. Based on the finite-time stability theory, a control law is proposed to realize finite-time chaos synchronization for the unified chaotic system with uncertain parameters. The controller is simple, robust and only part parameters are required to be bounded. Simulation results for the Lorenz, Lü and Chen chaotic systems are presented to validate the design and the analysis.

1. Introduction

Chaos synchronization has been a hot topic since the pioneering work of Pecocra and Carroll [1]. It can be applied in various fields such as chemical reactors, power converters, biological systems, information processing, secure communication [2–4], etc. Until now, a wide variety of approaches have been proposed for the synchronization of chaotic systems which include adaptive control [5,6], observer-based control [7], sliding mode control [8], backstepping control [9], active control [10], nonlinear control [11], control Lyapunov function method [12], and so on.

As time goes on, more and more people began to realize the important role of synchronization time. To attain fast convergence speed, many effective methods have been introduced and finite-time control is one of them. Finite-time synchronization means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [13].

In [14], Feng et al. proposed finite-time synchronization of two chaotic systems by using a terminal sliding mode controller and they adopted the Genesio chaotic system as their example. If the system has mismatched uncertainties, the state of the system can be driven to a small neighborhood of the equilibrium in a finite-time. In [15], Gilles et al. proposed finite-time global chaos synchronization for piecewise linear maps and discrete chaotic systems. In [16], Li and Tian mainly adopted the controller $u = -k\mathrm{sign}(x)|x|^{\alpha}$ to realize finite-time synchronization of some chaotic systems. In this paper, we present a con-

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tinuous controller to realize finite-time synchronization of the unified system with uncertain parameters. The controller is robust to parameter uncertainties and simple to be constructed.

2. Preliminary definitions and lemmas

Finite-time synchronization means that the state of the slave system can track the state of the master system after a finite-time. The precise definition of finite-time synchronization is given below.

Definition 1. Consider the following two chaotic systems:

$$\begin{aligned} \dot{x}_{\mathrm{m}} &= f(x_{\mathrm{m}}), \\ \dot{x}_{\mathrm{s}} &= h(x_{\mathrm{m}}, x_{\mathrm{s}}), \end{aligned} \tag{1}$$

where x_m, x_s are two n-dimensional state vectors. The subscripts 'm' and 's' stand for the master and slave systems, respectively. $f : R^n \to R^n$ and $h : R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t \to T} \|x_{\mathrm{m}} - x_{\mathrm{s}}\| = 0,$$

and $\|x_{\mathrm{m}} - x_{\mathrm{s}}\| \equiv 0$, if $t \geqslant T$, then synchronization of the system (1) is achieved in a finite-time.

Lemma 1 [14]. *Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \leqslant -cV^{\eta}(t) \quad \forall t \geqslant t_0, \; V(t_0) \geqslant 0, \tag{2}$$

where $c > 0, 0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), \quad t_0 \leqslant t \leqslant t_1, \tag{3}$$

and

$$V(t) \equiv 0 \quad \forall t \geqslant t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof. Consider the following differential equation:

$$\dot{X}(t) = -cX^{\eta}(t), \quad X(t_0) = V(t_0). \tag{6}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution of Eq. (6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t - t_0). \tag{7}$$

Therefore, from the comparison Lemma [18], one obtains

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), \quad t_0 \leqslant t \leqslant t_1 \tag{8}$$

and

$$V(t) = 0 \quad \forall t \geqslant t_1$$

with t_1 given in (5). □

Lemma 2 [17]. *When a, b and $c < 1$ are all positive numbers, the following inequality holds:*

$$(a+b)^c \leqslant a^c + b^c. \tag{9}$$

This result is quite straightforward and the proof is omitted here.

3. Main results

A chaotic system is extremely sensitive to its initial condition and tiny variations of parameters. In practical situation, the system is disturbed by parameter variation and this variation cannot be exactly known in priori. The effect of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization of systems with uncertainties. In this section, we first discuss finite-time synchronization of the determined unified system. Then, we turn the problem to the system with uncertain parameters.