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Stabilizing unstable fixed points of discrete chaotic systems via quasi-sliding mode method

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Abstract

The problem of stabilizing unstable fixed points of nonlinear discrete chaotic systems, subjected to bounded model uncertainties, is investigated in this article. The theory is then generalized to include any *d*th-order fixed point of the system. To design a suitable controller, the theory of quasi-sliding mode control is modified and applied. Sufficient conditions for the convergence of the control algorithm are theoretically derived and it is shown that the error trajectories converge toward a bounded region around zero where the measure of the steady-state error band depends on the magnitude of the system uncertainties. As a case study, the proposed method is applied to the Henon map to stabilize its first, second, and fourth-order unstable fixed points. Simulation results show the high performance of the control technique in quenching the chaos in the presence of uncertainties.

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1. Introduction

Control of nonlinear systems has always been of great interest due to their applications in real mechanical and electrical systems. Several control theories are developed right now for nonlinear continuous systems [1,2], while discrete systems are not studied that much. Of course, there exists some material in nonlinear control texts, but usually linear models are studied therein [3,4].

The fact that most of the continuous control schemes are implemented on a digital device has made many scientists study the behavior of discrete nonlinear systems. On the other hand, recently many economic and social systems are being discussed which are discrete in their nature. In addition, there exist some cases in

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which a discrete map, i.e. the Poincare map, can be obtained by intersecting the system flows by a hyper surface. A well-known example of this kind is the bouncing ball problem where the continuous dynamics of the ball is accompanied with the discrete impact phenomenon. Such problems (recently known as hybrid dynamical systems [5]) are another case where discrete control theories may be of great interest.

Chaos is a common phenomenon in nonlinear discrete dynamical systems [6]. The first documents in control of discrete chaotic systems return to 1990s. Ott et al. [7] proposed an innovative scheme to control chaos in which a small parameter perturbation converts the chaotic attractor to an attractor with large number of attracting periodic motions (see [8] for extensions of this method). In addition, many nonlinear and adaptive control methods are applied for chaos elimination in discrete systems. In [9], an adaptive model-independent method is applied to low-dimensional chaotic systems. Experimental control of chaos using the concept of stable and unstable manifolds of fixed points is also performed and appears in [10]. The indirect adaptive control of discrete chaotic systems based on feedback linearization is studied in [11].

Synchronization of chaotic systems with structure or parameter mismatches is discussed in [12] where a feedback control for synchronizing two chaotic systems is proposed based on sliding mode control design. A delayed feedback control method using nonlinear estimation of stabilized orbits is introduced in [13] to overcome the restrictions of Pyragas method [14] in comparison to the OGY method. Some limitations of the delayed feedback control is described in [15].

In continuous time chaotic systems, sliding mode control is an effective method even in the presence of bounded uncertainties [16,17]. On the other hand, there are a few works which have modified and applied sliding mode control to discrete systems. Corrandini and Orlando [18] deal with the problem of trajectory tracking for a wheeled mobile robot subjected to parametric uncertainties. The sliding mode control is implemented in discrete time to ensure its applicability and robustness. The work is accompanied with simulation and experimental results. In [19], a neuro-adaptive control is designed for a class of nonlinear discrete time systems using sliding mode control. Munoz and Sbarbro [20] described the necessary conditions that should be met and a control input satisfying the quasi-sliding mode conditions. Their method, although capable of handling modeling errors, needed some rigorous conditions to ensure the boundedness of the modeling errors. They proposed to use an artificial neural network to solve this problem.

The concept of sliding mode or quasi-sliding mode is not as clear in discrete problems as in continuous ones. Milosavjlevioc [21] proposed the condition $S_k(S_{k+1} - S_k) < 0$ for discrete-time systems to converge to the sliding surface. It is easy to verify that this condition is not sufficient to guarantee convergence to the sliding surface. Sarpturk et al. [22] considered that a necessary and sufficient condition for the existence of a discrete-time sliding mode to occur is $|S_{k+1}| < |S_k|$. This condition guarantees that the system representative point moves toward the switching surface. However, there is no guarantee that the switching surface will be reached or crossed. Furuta [23] proposed another convergence condition for discrete-time sliding mode, which was analogous to the condition of Sarpturk. Utkin and Drakunov [24] established that some sliding manifold must be reached in a finite time interval and, after reaching the sliding manifold, the system representative point must be confined to that region to ensure the existence of discrete-time sliding mode. Gao et al. [25] explained that discrete-time sliding mode systems should have the following three attributes:

(a) The error trajectories cross the switching surface in a finite time interval.
(b) Once the switching surface has been crossed, the error trajectories exhibit a zigzag motion around it.
(c) In the zigzag motion, the error remains in a certain bound.

In this paper, the method introduced in the work of Muñoz and Sbarbro [20] is modified for application to discrete chaotic systems. The main objective of this paper is to stabilize the unstable fixed points – of any order – in a chaotic system subjected to some bounded modeling uncertainties. To do so, a stable sliding surface is considered in terms of the past tracking errors. The control input required to make this sliding surface stable is designed and it is verified that subjected to such control signal, all the three conditions described above are satisfied; hence, making the sliding mode (or quasi-sliding mode) stable. Finally, the method is applied to a chaotic nonlinear system, namely the Henon map. Stabilization of its first, second, and fourth-order periodic orbits is performed in the presence of uncertainties in the system model. Simulation results show the high performance of the control scheme.