

Effects of seasonal growth on ratio dependent delayed prey predator system

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Abstract

The Beddington–DeAngelis ratio dependent prey predator model with time delay has been discussed. The existence of Hopf bifurcation has been established. The numerical simulations have shown that seasonal growth and delay can give rise to variety of attractors including periodic, quasi-periodic as well as chaotic oscillations. The degree of complexity in the system increases with increase in magnitude of delay, or frequency of seasonal variation. The model parameters involved in functional response can also affect the complexity of the system.

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1. Introduction

The construction and study of models for the population dynamics of predator–prey systems have been of interest to theoretical ecologists since the famous Lotka–Volterra equations. Many complex models for two or more interacting species have been proposed by taking into account the behavior of species due to socio-environmental factors. A most crucial element in prey predator models is the “functional response” or “trophic function”, the function that describes the number of prey consumed per predator per unit time for given quantities of prey and predators. The models incorporating effects of crowding, age structure, time delay, etc. have been reviewed [1,3,4].

The ratio dependent models are free of paradox of enrichment and that of biological control [5]. These models are criticized due to presence of singularity and associated difficulties in mathematical analysis of the model. It is known that the Beddington–DeAngelis form of functional response has some of the same qualitative features as the ratio dependent form but avoids singular behaviors of the ratio dependent models at low densities which have been the source of controversy [7].

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Factors that introduce time lags may include age structure of the population (influencing the birth and death rates), maturation periods (thresholds), feeding times, reaction times, food storage and hunger coefficients in predator–prey interactions. Models incorporating time delays in diverse biological models are extensively studied by MacDonald [9], Gopalsamy [2], Beretta and Kuang [5], Cushing [6], Song [13], Wang [14] and Kuang [4].

The population communities are imbedded in periodically varying environments. Therefore, it is quite natural to identify the functional role that play on the behavior of population communities. The basic problem is to understand the relationship between the magnitude of the seasonal variations and the complexity of the system. The first contribution on the subject is by Inoue and Kamifukumoto [8] who varied the intrinsic growth rate of the prey and focused their discussion on the influence of the frequency of the forcing function. However, due to such variation, the two-dimensional non-autonomous systems are reported to have complex dynamical behavior [8,11,12].

Naturally, more realistic and interesting models of population interactions should take into account both the seasonality of the changing environment and the effects of time delay. Therefore, it is interesting and important to study the non-autonomous delayed predator–prey model with a ratio dependent functional response.

In this paper, the effects of seasonality in delayed prey predator system with Beddington–DeAngelis functional response is discussed. The effects of time delays due to conversion of food into predator growth [5,13] and negative feedbacks of prey species to the growth of species itself [10] are included in the model.

2. Model

Consider prey predator model using Beddington–DeAngelis functional response with time delay. Let $u_1(t)$ and $u_2(t)$ be the density of prey and predator respectively. The delay terms occur in growth as well as in interaction terms. For this, it is assumed that the prey takes time τ_1 to convert the food into its growth [10], whereas the predator takes time τ_2 for the same [5,13]. Accordingly, the delayed prey predator model can be expressed as

$$\begin{aligned} \frac{du_1(t)}{dt} &= u_1(t)[1 - u_1(t - \tau_1)] - \frac{a_1 u_1(t) u_2(t)}{a + u_1(t) + b u_2(t)} \\ \frac{du_2(t)}{dt} &= d_1 u_2(t) \left[-d + \frac{u_1(t - \tau_2)}{a + u_1(t - \tau_2) + b u_2(t - \tau_2)} \right] \end{aligned} \tag{1}$$

Assume all the parameters in a model take positive values i.e. $a_1 > 0, d > 0, d_1 > 0, a > 0, b > 0$. Further, the following initial functions are associated with the system (1):

$$\begin{aligned} &\left. \begin{aligned} u_{10}(\theta) &= \psi_1(\theta) \geq 0 \\ u_{20}(\theta) &= \psi_2(\theta) \geq 0 \end{aligned} \right\} \theta \in [-\tau, 0], \quad \tau = \max\{\tau_1, \tau_2\} \\ \psi &= (\psi_1, \psi_2) \in C([-\tau, 0], \mathbb{R}_+^2), \quad \mathbb{R}_+^2 = \{(u_1, u_2) : u_1 \geq 0, u_2 \geq 0\} \end{aligned}$$

3. Local stability analysis and Hopf bifurcation

To simplify the analysis, it is assumed that both the delays are of equal magnitude, i.e. $\tau_1 = \tau_2 = \tau$. The system has three equilibrium points: two boundary equilibrium points $E_1(0, 0)$ and $E_2(1, 0)$, and a non-trivial equilibrium point $\tilde{E}(\tilde{u}_1, \tilde{u}_2)$.

The boundary equilibrium points always exist. The existence of non-trivial equilibrium point is subject to the conditions under which the positive solutions of the following quadratic equations are possible:

$$\tilde{u}_1^2 + \alpha_1 \tilde{u}_1 + \beta_1 = 0, \quad \alpha_1 = \frac{(a_1 - da_1 - b)}{b}, \quad \beta_1 = -\frac{ada_1}{b} \tag{2}$$

$$\tilde{u}_2^2 + \alpha_2 \tilde{u}_2 + \beta_2 = 0, \quad \alpha_2 = \frac{a_1(d - 1)^2 + b(2ad + d - 1)}{b^2 d}, \quad \beta_2 = \frac{a(ad + d - 1)}{b^2 d} \tag{3}$$

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