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DQM large amplitude vibration of composite beams on nonlinear elastic foundations with restrained edges

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Abstract

This paper presents an efficient and accurate differential quadrature (DQ) large amplitude free vibration analysis of laminated composite thin beams on nonlinear elastic foundation. Beams under consideration have elastically restrained against rotation and in-plane immovable edges. Elastic foundation has cubic nonlinearity with shearing layer. We impose the boundary conditions directly into the governing equations in spite of the conventional DQ method and without any extra efforts. A direct iterative method is used to solve the nonlinear eigenvalue system of equations after transforming the governing equations into the frequency domain. The fast rate of convergence of the method is shown and their accuracy is demonstrated by comparing the results with those for limit cases, i.e. beams with classical boundary conditions, available in the literature. Besides, we develop a finite element program to verify the results of the presented DQ approach and to show its high computational efficiency. The effects of different parameters on the ratio of nonlinear to linear natural frequency of beams are studied.

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Keywords: Large amplitude; Free vibration; Angle-ply laminated beams; DQM; Elastic foundation; Elastically restrained edges

1. Introduction

Composite laminated beams with high stiffness and strength to weight ratio are increasingly used in many engineering structures such as commercial and military aircrafts. In most conditions of severe environment, the laminated beams can undergo large amplitude vibrations. However, the nonlinear free vibration analyses of composite laminated beams have attracted little attention of researchers [1–3].

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Nomenclature

A_{ij}	extensional stiffness of laminate
$A_{ij}^{x(r)}$	weighting coefficient of the rth order
A_{ij}^{x}	weighting coefficients of the first-order derivative
B_{ij}^{x}	weighting coefficients of the second-order derivative
b	width of the beam
D_{ij}	bending stiffness of laminate
D_{ij}^{x}	weighting coefficients of the fourth-order derivative
E_{ij}	Young's modulus of lamina
G_{ij}	shear modulus of lamina
h	total thickness of beam
I_{oo}	inertia of the lamina
k_1, k_2	linear and nonlinear coefficients of elastic foundation
k_g	coefficient of shearing layer elastic foundation
k_{ti}	coefficients of rotational stiffness at the edges of the beam ($i = 1, N_x$)
K_1, K_2	non-dimensional linear and nonlinear elastic foundation coefficients $\left[= \left(k_1, \frac{k_2}{h^2}\right)\frac{D_{11}}{L^4}\right]$
K_{Lt}, K_{Rt}	non-dimensional coefficient of rotational stiffness at the left and right edges of the beam $[= (k_{t1}, k_{tN_x})D_{11}/L]$
K_g	non-dimensional coefficients of shearing layer elastic foundation $k_g D_{11}/L$
M_{xx}	bending moment
N_e	number of elements for finite element method (FEM)
N_x	number of grid points in x-directions
N_{xx}	in-plane normal force resultant in x-directions
$(N_{xx})_{ij}$	discretized in-plane normal force resultant in x-directions
n_x	the x-component of unit normal vector to an arbitrary edge of the beam
u, v, w	displacement component in the x, y and transverse direction of a point on mid-plane of beam, respectively
$\{u\}_d, \{w\}_d$	vector of x and z-components of the displacement at the domain grid points, respectively
W	amplitude of transverse displacement
x, y, z	the Cartesian coordinate variables
λ_i	nonlinear to linear frequency ratio of laminated beam ($=\omega_{i\mathrm{NL}}/\omega_{i\mathrm{L}}$)
K_i^x	secondary degrees of freedom at the edges of beam $\left[= \left.\frac{\partial^2 w}{\partial x^2}\right\vert_{x=x_i}\right]$
ρ_i	mass density of the lamina
υ_{ij}	Poisson's ratio of lamina
ω^r	frequency at rth iteration

Kapania and Raciti [1] proposed a two-noded Timoshenko beam element with 10 degrees of freedom per node to study the nonlinear free vibrations of laminated composite beams by employing the perturbation method. Ganapathi et al. [2] considered the large amplitude free vibration analysis of cross-ply laminated straight and curved beams using spline element method. Patel et al. [3] presented a three-noded shear flexible beam element to study the free vibration and post-buckling of laminated orthotropic beams resting on a two parameters elastic foundation (Pasternak type). In all of these works, beams with classical boundary conditions were examined and also only the fundamental nonlinear frequency was given.

To the best authors knowledge there is no work available in the literature on the large amplitude free flexural vibration analysis of laminated composite beams on three parameters nonlinear elastic foundation with elastically restrained against rotation edges.

Differential quadrature method (DQM) is a relatively new numerical technique in structural analyses [4–16]. It was employed for some nonlinear free vibration of isotropic beams and orthotropic plates