Available online at www.sciencedirect.com
ScienceDirect
Communications in Nonlinear Science and Numerical Simulation 13 (2008) 637–644
Communications in Nonlinear Science and Numerical Simulation
www.elsevier.com/locate/cnsns

Lag synchronizing chaotic system based on a single controller

Dongxiao Wang [a,b], Yongli Zhong [a], Shihua Chen [a,*]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China
[b] Department of Mathematics and Physics, Zhengzhou Institute of Aeronautical Industry Management, Zhengzhou 450015, PR China

Received 7 April 2006; accepted 14 May 2006
Available online 10 July 2006

Abstract

Lag synchronization of chaotic system is investigated. Three kinds of schemes are proposed to lag synchronize Chen chaotic system. All the three schemes need only a single controller to realize lag synchronization. Especially in the last two schemes, only one state variable is contained in controller, which is of important significance on using chaos lag synchronization for applications. Finally numerical simulations are provided to show the effectiveness of the developed methods.

PACS: 05.45.−a

Keywords: Lag synchronization; Chaos; Chen chaotic system; A single controller

1. Introduction

Over the last three decades, chaos synchronization has become a popular research topic arousing interests of physical scientists and electrical engineers ([1–7], and the references therein). Synchronization strategies have great potential applications in several areas such as secure communication, biological oscillations and animal gaits. Synchronization means that $\lim_{t\to\infty}|y(t) - x(t)| = 0$, where $x(t)$ and $y(t)$ are the states of the master and slave systems, respectively. It has been shown that the synchronization behavior can be induced either by coupling the systems or by forcing them. However, in engineering applications, time delay always exists. For example in the telephone communication system, the voice one hears on the receiver side at time t is the voice from the transmitter side at time $t - \tau$. So, strictly speaking, it is not reasonable to require the slave system to synchronize the master system at exactly the same time. Therefore, recently, much attention has been given to the lag synchronization [8–10], in which the state of the slave system at time t is asymptotically

* Corresponding author. Tel.: +86 2768775159.
E-mail addresses: cshwhu@163.com (D. Wang), shcheng@whu.edu.cn (S. Chen).

1007-5704/$ - see front matter © 2006 Published by Elsevier B.V.
doi:10.1016/j.cnsns.2006.05.005

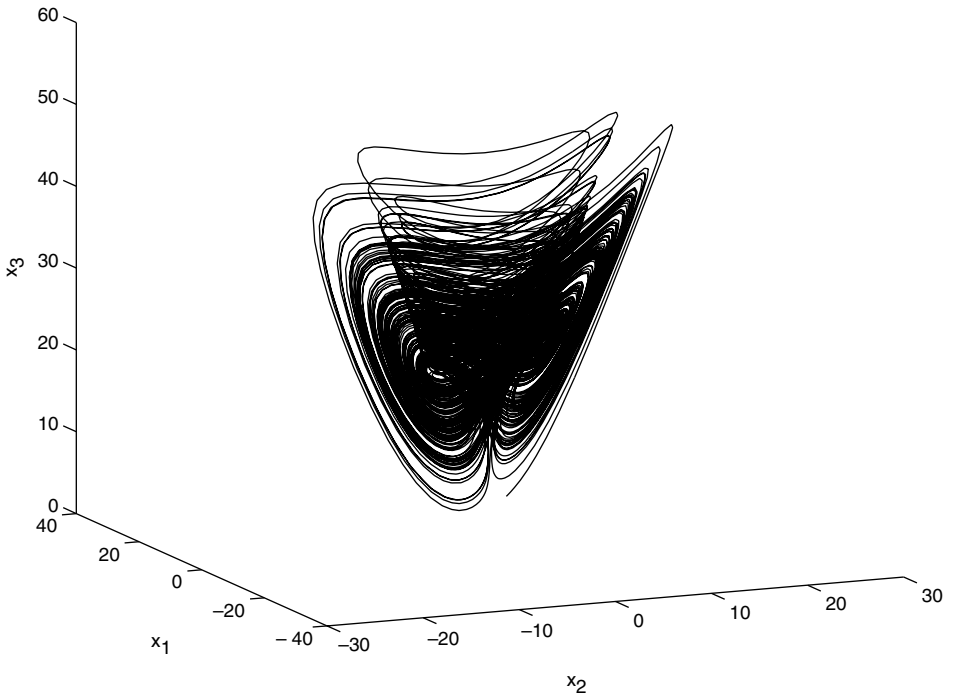

Fig. 1. Chaotic behavior of Chen system.

synchronous with the master system at time $t-\tau$, namely, $\lim_{t\to\infty}|y(t)-x(t-\tau)|=0$, where $x(t)$ and $y(t)$ are the states of the master and slave systems, respectively.

In this paper, we propose three kinds of schemes to lag synchronize Chen chaotic system. All the three schemes need only a single controller to realize lag synchronization. Especially in the last two schemes, only one state variable is contained in controller, which is of important significance on using chaos lag synchronization for applications. Finally numerical simulations are provided to show the effectiveness of the developed methods.

2. Chen chaotic system

Chen chaotic system is described by the following ordinary differential equation [11]

$$\begin{aligned}\dot{x}_1 &= a(x_2-x_1),\\ \dot{x}_2 &= (c-a)x_1-x_1x_3+cx_2,\\ \dot{x}_3 &= x_1x_2-bx_3,\end{aligned} \tag{1}$$

where a, b and c are the system parameters. If we choose $a=35$, $b=3$, $c=28$, the chaotic behavior is shown in Fig. 1.

Chen chaotic system is quite similar to Lorenz chaotic system, but they are not equivalent topologically, namely, there is no diffeomorphism that can transfer one to another. Moreover, Chen attractor is seemingly more complicated than Lorenz attractor in terms of dynamics since the former has prominent three-dimensional features in the phase-space [12]. This feature may be useful because a more complex chaotic system may lead to more secure communications.

3. Lag synchronizing Chen chaotic system

In this section, we first propose a systematic design procedure to lag synchronize Chen chaotic system based on back-stepping procedure [13]. This method needs only a single controller to realize lag synchronization. The aim is to design a controller U_1 such that the controlled Chen chaotic system