Engineering Fracture Mechanics 78 (2011) 2220–2234

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A three-dimensional eigenfunction expansion approach for singular stress field near an adhesively-bonded scarf joint interface in a rigidly-encased plate

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ARTICLE INFO

Article history:
Received 30 July 2010
Received in revised form 8 April 2011
Accepted 12 April 2011
Available online 19 April 2011

Keywords:
Scarf joint
Bi-material interface
Three-dimensional
Stress singularity
Adhesive layer
Fixed edge singularity

ABSTRACT

A three-dimensional eigenfunction expansion approach for determination of the singular stress field in the vicinity of an adhesively-bonded scarf joint interface in a plate, with its top and bottom surfaces being encased, fully or partially, between infinitely rigid blocks is presented. The plate is subjected to extension/bending (mode I) and in-plane shear/twisting (mode II) far-field loading. Both the adhesive layer and plate materials are assumed to be isotropic and elastic. The boundary conditions that are prescribed on the end-faces (free, fixed and lubricated) of the plate as well as those, prescribed at the bottom or top surface of the scarf-bonded plate on either side of the interface between the plate and adhesive layer materials (fixed-fixed, free-fixed and fixed-free), are exactly satisfied. Numerical results include the dependence of the lowest eigenvalue (or most severe stress singularity) on the wedge aperture angle of the plate material. Variation of the same with respect to the shear moduli ratio of the constituent plate and adhesive layer materials is also an important part of the present investigation. Hitherto unobserved interesting and physically meaningful conclusions in regards to the fixed edge singularity and delamination type flaw sensitivity of an adhesively bonded plate surface are also presented. Finally, hitherto unavailable results, pertaining to the through-width variations of stress intensity factors corresponding to symmetric and skew-symmetric sinusoidal loads that also satisfy the boundary conditions on the end-faces of the adhesively bonded plate, in the vicinity of the scarf joint interface, under investigation, bridge a longstanding gap in the bonded joint stress singularity/fracture mechanics literature.

1. Introduction

Panels, adhesively bonded by means of scarf joints, are widely used in modern structural technology, because such joints are more efficient in load transfer and provide higher strengths than their lap-shear counterparts [1–3]. Figs. 1 and 2 show the schematics of a scarf joint. Because of its intrinsic characteristics, any bonded joint acts as a stress raiser, characterized by highly three-dimensional stress state coupled with high stress gradient and straight face singularity, of which the well-known two-dimensional free edge singularity is a special case [4,5]; they are ultimately responsible for delamination type

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doi:10.1016/j.engfracmech.2011.04.009

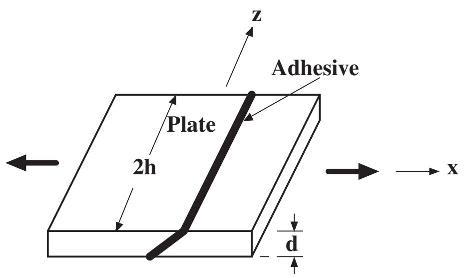

Fig. 1. Schematic of an adhesively-bonded scarf joint.

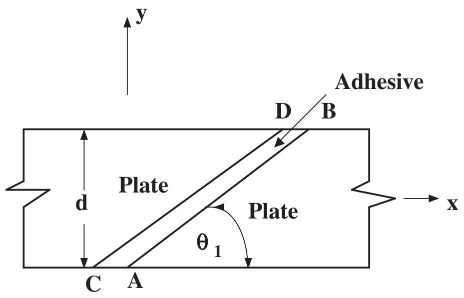

Fig. 2. Geometry of a cross-section of a scarf-bonded plate.

joint failures [4]. A realistic analysis of these adhesively bonded materials must account for these adverse effects in order to insure integrity and design safety of structures.

The most critical element in the scarf joint is the thin adhesive layer. When an adhesively bonded beam or plate is used in structural applications such as aircraft, spacecraft, marine, automobile and so on, it is generally connected to other structural members, and may sometimes be encased by much more rigid structural elements, which may be idealized as rigid blocks. A fully or partially clamped face of a bi-material plate has recently been shown to exhibit stress singularities [6] similar to their free edge (free end-face in three-dimension) counterpart. While the latter class of singularities has been extensively studied in the two-dimensional literature, the former is conspicuous by its near-absence. The primary motivation of the present investigation stems from the need for understanding the three-dimensional singular stress field in the vicinity of a scarf-bonded joint interface between the plate material and the adhesive layer, encased fully or partially between rigid blocks. A review of the literature reveals a total absence of such investigations.

Three-dimensional bi-material wedges of semi-circular cross-sections (or bi-material half-spaces of finite thickness) are relevant to the analysis of scarf joint failure, caused by the presence of the stress singularity at or near a scarf-bonded joint interface between the plate material and the adhesive layer. Asymptotic behavior of two-dimensional stress fields at the tips of bi-material wedges made of isotropic materials and subjected to far-field Modes I, II or III loading, has been studied extensively in the literature; see e.g., Refs. [7,8]. The mathematical difficulties posed by the three-dimensional bi-material wedge problems are substantially greater than their two-dimensional counterparts (to start with the governing PDE's are much more complicated). A three-dimensional eigenfunction approach has recently been developed by Chaudhuri and Xie [9], and Chaudhuri [10] for understanding the singular stress behavior in the vicinity of a point located at the front of a crack and an anticrack, respectively. This has been extended to determine the asymptotic stress fields in the neighborhood of points located at the fronts of homogeneous and bi-material pie-shaped wedges [6], and their special cases of bi-material interface cracks [11] and free edges/end-faces [12]. The problems of three-dimensional asymptotic stress field in the neighborhood of the front of bi-material pie-shaped wedges of symmetric and unsymmetric geometrical configurations (with respect to the bi-material interface) and subjected to antiplane shear far-field loading have also recently been solved by Xie and Chaudhuri [13], and Chiu and Chaudhuri [14], respectively. More recently, solutions to the problems of three-dimensional asymptotic stress fields in the vicinity of the front of unsymmetric fiber-matrix pie-shaped wedges formed as a result of matrix cracking or fiber break and subjected to Mode 1/II far-field loading (extension/bending and in-plane shear/twisting) have also become available in the literature [15]. It may also be noted here that the above three-dimensional eigenfunction expansion technique has also been utilized to compute the asymptotic stress fields in the vicinity of fronts of penny shaped cracks/anticracks weakening homogeneous and bi-material media [16,17], and thus establishing conceptual similarity of this class of problems with their through-thickness counterparts. In addition, the three-dimensional singular stress fields near a partially debonded cylindrical rigid fiber [18], and in the vicinity of the circumferential tip of a fiber-matrix interfacial debond [19,20] have also been derived using the same afore-mentioned three-dimensional eigenfunction