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Towards optimization of crack resistance of composite materials by adjustment of fiber shapes

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ABSTRACT

We investigate the evolution and propagation of cracks in 2-d elastic domains, which are subjected to quasi-static loading scenarios. In addition to the classical variational formulation, where the standard potential energy is minimized over the cracked domain under physical conditions characterizing the behavior of the material close to the crack (e.g. non-penetration conditions), we include a 'cohesive traction term' in the energy expression. In this way we obtain a mathematically concise set of partial differential equations with non-linear boundary conditions at the crack interfaces. We perform a finite element discretization using a combination of standard continuous finite elements and so-called cohesive elements. During the simulation process cohesive elements are adaptively inserted at positions where a certain stress bound is exceeded. In our numerical studies we consider domains consisting of a matrix material with fiber inclusions. Beyond pure crack path simulation, our ultimate goal is to determine an optimal shape of the fibers resulting in a crack path that releases for a given load scenario as much energy as possible without destroying the specimen completely. We develop a corresponding optimization model and propose a solution algorithm for the same. The article is concluded by numerical results.

1. Introduction

Solving crack problems in fracture mechanics has occupied researchers for many decades in order to be able to quantify and predict the behavior of cracked structures under service conditions. Theoretical foundations of the classical theory of brittle fracture in solids are outlined in the works of Griffith [1], Irwin [2] and Barenblatt [3]. Later studies consider the evolution problem of brittle fracture based on material forces acting at the crack tip singularities [4,5] and refer to Eshelby [6] and Rice [7]. Rigorous mathematical investigations of crack problems like variational approaches (see for example [8,9]) or methods, where an asymptotic formula is obtained for the total energy increment during quasi-static crack growth [10] have been developed much later than the first numerical methods to simulate cracking processes. The early finite element studies like in [11] have extensively used the nodal release procedure to simulate crack growth. The lack of a material length scale using this method leads to strong dependence of the finite element results on the size of the elements near the crack tip. One answer is given by meshless methods like the element-free Galerkin method [12] or the extended finite element method

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Nomenclature

$\boldsymbol{A}$	stiffness matrix with penalization term included
$\boldsymbol{A^r}$	matrix with B-spline basis functions
b_i	basis spline function
B_r	design curve for fiber boundary r
$\boldsymbol{c}_{ijkl}$	stress tensor ($i, j, k, l = 1, 2$)
$\boldsymbol{C}$	constraint functionals
da	infinitesimal length element
$\boldsymbol{d_i^r}$	coordinate vector of the ith control point on Γ_r
$d\Omega$	infinitesimal area element
$\boldsymbol{D}$	matrix composed of all $\boldsymbol{D^r}$ of all Γ_r, $r = 1,\ldots,N$
$\boldsymbol{D^r}$	matrix composed of coordinates of all control points associated with Γ_r
$\boldsymbol{e}_{kl}$	strain tensor ($k, l = 1, 2$)
E	Young's modulus
$\boldsymbol{f}$	external traction
$\boldsymbol{f_{coh}}$	nonlinear cohesive traction term in discretized system
$\boldsymbol{F}$	finite matrix whose columns in $\mathbb{R}^s$ represent search directions
$\boldsymbol{F_k}$	columns of $\boldsymbol{F_k}$ chosen from $\boldsymbol{F}$
G	energy release rate
$\boldsymbol{H}$	matrix representing the linear mapping from $\boldsymbol{D}$ on $\boldsymbol{p_{NB}}$
K	set of admissible displacements
L	feasible region for optimization variables (NOMAD)
$m_r + 1$	number of finite element nodes on fiber boundary Γ_r
M	grid for pattern search
n	outer unit normal vector to $\partial\Omega$
$n_r + 1$	number of control points
N	number of fibers
$\boldsymbol{p_B^r}$	finite element nodes lying on fiber boundary Γ_r
$\boldsymbol{p_{NB}}$	all non-boundary finite element nodes
P	potential energy
P_k	POLL set
q_i^r	optimization variable corresponding to control point $\boldsymbol{d_i^r}$
S	surface energy
t_i^r	parameter for B-spline
$\boldsymbol{t}$	cohesive traction
$\boldsymbol{T^r}$	parameter vector for B-spline
T_ϵ	penalized total energy functional
$\boldsymbol{u}$	displacement field
$\boldsymbol{v}$	test function
V	admissible set of test functions
W	fracture energy
Γ_{ci}	ith crack part of crack Γ_C
$\Gamma_{e_{\max}}$	element side with maximal stress
Γ_r	boundary of fiber r
Γ_C	crack
Γ_D	Dirichlet boundary
Γ_N	Neumann boundary
$\boldsymbol{\delta}$	crack opening
ϵ	penalty parameter
$\mathscr{E}$	space of admissible designs for fiber shapes
$\boldsymbol{\nu}$	unit vector normal to Γ_C
ν_p	Poisson ratio
σ_c	critical stress
$\boldsymbol{\sigma}_{ij}$	stress tensor ($i, j = 1, 2$)
$\boldsymbol{\tau}$	unit vector tangential to Γ_C
ω_r	domain of fiber r
Ω	computational domain
Ω_0	computational domain without crack
$(\cdot)_c$	critical cohesive parameter
$(\cdot)_{c\nu}$	critical cohesive parameter referring to normal crack opening