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Characteristics of dynamic brittle fracture captured with peridynamics

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ABSTRACT

Using a bond-based peridynamic model, we are able to reproduce various characteristics of dynamic brittle fracture observed in experiments; crack branching, crack-path instability, asymmetries of crack paths, successive branching, secondary cracking at right angles from existing crack surfaces, etc. We analyze the source of asymmetry in the crack path in numerical simulations with an isotropic material and symmetric coordinates about the pre-crack line. Asymmetries in the order of terms in computing the nodal forces lead to different round-off errors for symmetric nodes about the pre-crack line. This induces the observed slight asymmetries in the branched crack paths. A dramatically enhanced crack-path instability and asymmetry of the branching pattern are obtained when we use fracture energy values that change with the local damage. The peridynamic model used here captures well the experimentally observed successive branching events and secondary cracking. Secondary cracks form as a direct consequence of wave propagation and reflection from the boundaries.

1. Introduction

1.1. Literature review of dynamic brittle fracture

In experiments on dynamic brittle fracture a variety of phenomena are observed (see [1–5]): crack branching (bifurcation of crack), crack-path instability, successive branching events, secondary (or "circumferential") cracking (emerging along the primary cracks at $\sim 90^\circ$ angles), asymmetries of crack paths, etc. In a brittle material, cracks propagate rapidly and may curve or split into two or more branches. Under very high states of stress, a propagating crack can generate river-delta crack patterns [1,2]. Cracks propagating in brittle materials can generate fragmentation, which is often the result of a succession of multiple branching events from what was initially a single crack. Moreover, impact experiments on brittle targets reveal radial cracks and circumferential cracks, normal to the radial cracks [6]. In the experiments conducted in [3], waves are shown to generate cracks that propagate normal to the existing crack paths. Interestingly, experimental results on dynamic fracture show that crack paths are not symmetric even if loading conditions and the geometry are (see [1–5]). In this paper we use the peridynamic model to compute solutions to dynamic brittle fracture problems and assess the effectiveness of the simulations in capturing the various characteristic features observed by experimentalists in dynamic brittle fracture events.

Significant efforts have been dedicated to simulate dynamic brittle fracture phenomena over the past several decades. Progress has been made in the last two decades (see, e.g., [7]) and several models are now able to capture some of the characteristics of dynamic brittle fracture. Nevertheless, predicting dynamic fracture in brittle materials is still an open problem and many of the experimentally-observed features of dynamic brittle fracture are not reproduced in simulations unless pre-inserted as special criteria in the model. Three main classes of models have been used to simulate dynamic crack propagation

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Nomenclature

$\boldsymbol{b}$	prescribed bo dy force intensity
c	micromodulus function
D	damage index
E	Young's modulus
$\boldsymbol{f}$	pairwise force function in the peridynamic bond
G_0	energy release rate
H_0	internal subregion
m	ratio between the horizon size and the grid spacing
s, s_0	relative elongation, critical relative elongation
t	time
$\boldsymbol{u}$	displacement vector
$\ddot{\boldsymbol{u}}$	acceleration vector
W	elastic strain energy density
$\boldsymbol{x}$	position vector
α, β, γ	damage stretch coefficients
δ	horizon
$\boldsymbol{\eta}$	relative displacement vector
ν	Poisson ratio
ρ	mass density
ω	micro-elastic potential
ξ	relative position vector

in brittle materials: atomistic models (e.g. [8,9]), lattice models (e.g. [10]), and continuum-based models (FEM and XFEM models [7,11–17]). One of the main difficulties of atomistic models in predicting the behavior of dynamic cracks in brittle materials is the size of the simulation: propagation of dynamic cracks is critically influenced by stress waves (e.g. waves reflected from the boundaries of the structure) and atomistic models would be required to simulate the entire structure to capture these waves, a task that currently is impossible to do. In atomistic simulations with absorbing boundaries [18], the crack branching angles are several times larger than the experimentally observed ones [2]. Lattice dynamics models produce ranges of "forbidden velocities" but these are not observed in experiments [10]. In continuum-based approaches, cohesive-zone models modify the local continuum mechanics equations and introduce an internal length-scale related to the cohesive-zone parameters ([7,11–17]). The cohesive-zone Finite Element Method (FEM) or XFEM require a damage criterion and tracking of the stresses around the crack-tip to decide when to insert a branching point at the tip of a crack. In the FEM, the crack advances along the element sides by separating elements from one another, hence the crack path is forced to follow the mesh geometry (see [11–13]). Consequently, the computed crack path deviates from the "correct" crack path and the elastic energy in the systems differs from the actual one. Mesh dependency is a related problem in cohesive-zone FEM-based methods and Zhou and Molinari [15] suggest special mesh refinement strategies to reduce it. The XFEM method ([7]) allows cracks to pass through the finite elements, with some increase in cost for performing integration over the cut elements. The crack path is usually tracked by a level-set function ([7]). With a branching criterion, the method can simulate crack branching in dynamic fracture but it does not predict the experimentally-observed crack propagation speeds unless the material's fracture energy values are modified by a significant factor (see [17]).

A new continuum model, peridynamics, originally designed for modeling dynamic fracture, has been introduced in [19]. Recent results obtained in [20,21] using the bond-based peridynamic model for the crack branching problem show that peridynamics correctly predicts important elements of dynamic crack propagation: the shape of the crack paths, the general profile of the crack propagation speed (similar to the experimental one reported in [22]), attempted and successful branching events (similar to those observed in [2]), and the relation between the way strain energy is delivered into the fracture zone and the evolution of the fracture process (as reported in the experiments in [3,5]). The peridynamic results (the crack propagation speed and the crack path) appeared to converge [20,21] once the horizon (the nonlocal parameter) reached sub-millimeter values. These values may be related to the characteristic length discussed in [2,4].

In this paper, we employ the bond-based peridynamic model, also used in [20,21], and show that the peridynamic model is capable of reproducing more of the dynamic brittle fracture characteristics observed in the experiments mentioned above. We address the issue of crack path stability and branching symmetry and the effects induced by the use of fracture energy that varies with the local damage values. We find the source for small asymmetries of the branched crack paths, in an otherwise perfectly symmetric (about the crack line) computational model, to be in the search algorithm. We obtain successive crack branching events when higher stress levels are reached at the tip of the crack before propagation begins. We also analyze secondary, circumferential-type cracking which develops subsequent to the propagation of the main (radial-type) cracks, as a result of stress waves reflected from the free surfaces. These cracks grow perpendicular to the main cracks.