Engineering Fracture Mechanics 78 (2011) 1182–1200

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

T-stress effects on steady crack growth in a thin, ductile plate under small-scale yielding conditions: Three-dimensional modeling

J.C. Sobotka, R.H. Dodds Jr. *

Department of Civil & Environmental Engineering, University of Illinois, Urbana, IL 61801, USA

ARTICLE INFO

Article history:
Received 24 June 2010
Received in revised form 6 November 2010
Accepted 24 November 2010
Available online 1 December 2010

Keywords:
T-stress
Constraint effects
Steady crack growth
Fracture mechanics
Non-dimensional scaling
Three-dimensional effects
Ductile void growth

ABSTRACT

The non-singular T-stress provides a first-order estimate of geometry and loading mode, e.g. tension *vs*. bending, effects on elastic–plastic, crack-front fields under mode I conditions. The T-stress has a pronounced effect on measured crack growth resistance curves for ductile metals – trends most computational models confirm using a two-dimensional setting. This work examines T-stress effects on three-dimensional (3D), elastic–plastic fields surrounding a steadily advancing crack for a moderately hardening material in the framework of a 3D, small-scale yielding boundary-layer model. A flat, straight crack front advances at a constant quasi-static rate under near invariant local and global mode I loading. The boundary-layer model has thickness B that defines the only geometric length-scale. The material flow properties and (local) toughness combine to limit the in-plane plastic-zone size during steady growth to at most a few multiples of the thickness (conditions obtainable, for example, in large, thin aluminum components). The computational model requires no crack growth criterion; rather, the crack front extends steadily at constant values of the plane-stress displacements imposed on the remote boundary for the specified far-field stress intensity factor and T-stress. The specific numerical results presented demonstrate similarity scaling of the 3D near-front stresses in terms of two non-dimensional loading parameters. The analyses reveal a strong effect of T-stress on key stress and strain quantities for low loading levels and less effect for higher loading levels, where much of the plastic zone experiences plane-stress conditions. To understand the combined effects of T-stress on stresses and plastic strain levels, normalized values from a simple void-growth model, computed over the crack plane for low loading, clearly reveal the tendency for crack-front tunneling, shear-lip formation near the outside surfaces, and a minimum steady-state fracture toughness for $T = 0$ loading.

1. Introduction

The elastic–plastic, ductile tearing process in thin structural components often leads to plastic-zone sizes comparable to at most a few multiples of the thickness, B, especially for alloys with high yield strength relative to toughness. During structural overloads or in laboratory tests, a sharp crack formed by fatigue (for example) blunts under plastic deformation and begins to extend by a ductile tearing mechanism. As the crack advances (Δa) through the plastic zone formed during the initial blunting and early growth stages, the tearing resistance (e.g. J) increases, but at a decreasing rate with continued Δa, as the crack extends through material that has already experienced high (triaxial) stress, plastic strain and damage (e.g., void nucleation-growth) [1]. The present study focuses on this fracture process where the material's (local) tearing

* Corresponding author. Tel.: +1 2173333276.
E-mail addresses: sobotka@illinois.edu (J.C. Sobotka), rdodds@illinois.edu (R.H. Dodds).

0013-7944/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfracmech.2010.11.018

Nomenclature

B	thickness
E	elastic modulus
J	value of J-integral
K_I	mode I stress intensity factor
$\overline{K}$	normalized stress intensity factor
T	elastic T-stress
$\overline{T}$	normalized T-stress
r_c	planar dimension of rectangular prism elements
r_{p0}	extent of plastic deformation ahead of crack front
h_{pw}	height for trailing plastic wake
u_i	displacement vector using indicial notation
ε_{ij}	strain tensor using indicial notation
σ_{ij}	stress tensor using indicial notation
σ_1	maximum principal stress
σ_2	intermediate principal stress
σ_3	minimum principal stress
$\bar{\varepsilon}^p$	equivalent plastic strain
σ_m	mean stress
σ_e	Mises stress
ρ	void radius
ζ	level of void growth expansion
ζ_{ref}	reference level of void growth expansion
(r,θ,Z)	cylindrical coordinate system with origin at crack front location
(X,Y,Z)	Cartesian coordinate system with origin at crack front location
μ	elastic shear modulus
ν	Poisson's ratio
σ_0	material yield stress
ε_0	material yield strain
SSY	small-scale yielding
3D	three-dimensional

toughness, combined with (comparatively) high yield strength, cannot sustain a plastic-zone size of more than a few thicknesses.

Continued crack advance occurs under globally plane-stress and essentially steady-state conditions when the in-plane dimensions of the structural component or laboratory specimen exceed many times the thickness and amount of crack extension. The immediate crack-front material over the mid-thickness region experiences elastic–plastic, near plane-strain conditions which quickly become elastic–plastic, plane stress and then simply linear-elastic, plane-stress conditions with increasing radial distance from the crack front. Aerospace applications with thin components (a few mm's) constructed from modern aluminum alloys (e.g., yield stress of 350 MPa, toughness of 40 MPa$\sqrt{\text{m}}$) readily create these conditions of three-dimensional, small-scale yielding (3D SSY) — see for example the laboratory fracture tests conducted in Newman et al. [2] and Seshadri et al. [3] on aluminum panels that exceeded 1 m in size.

In [4], we describe the implementation and application of a computational framework for steady-state, mode I crack growth obtained by extending the Dean and Hutchinson [5], streamline-integration methodology from plane-stress/strain conditions to three dimensions. For a straight crack front, the plate thickness, B, then provides a geometric length-scale in addition to deformation scales of plastic-zone size and crack-tip opening. This computational setting directly yields the steady-state solution (3D displacement, strain, and stress fields) for elastic–plastic crack growth on a fixed finite element mesh. The model requires no crack growth criterion to drive crack extension — steady crack advance occurs at the loading defined by the plane-stress, K_I displacements imposed on the remote boundary. Previous studies adopt the plane-stress/strain formulations of the elastic–plastic, steady-growth methodology to examine various features of the fracture process, including inertial effects [6], near-tip energy-balance requirements [7,8], constraint effects [9,10], and mixed-mode crack advance [11]. Other investigators employ the 3D SSY model successfully with a stationary crack to explore dimensional scaling effects of thickness [12], to compare computational with analytical void growth rates [13], and to investigate the crack-front blunting process in a finite deformation framework [14].

This study extends our 3D SSY framework in [4] for mode I, steady crack growth to include the potential effects of T-stress on the elastic–plastic, fracture process — examined for a stationary crack front by Yuan and Brocks [15] and Kim et al. [16], and for a crack advancing under cyclic loading by Roychowdhury and Dodds [17]. The linear-elastic T-stress provides a first-order estimate of geometry and loading mode (e.g., tension *vs*. bending) effects on near-front, elastic–plastic fields. The far-field loading in the 3D SSY model becomes the combined plane-stress displacements generated by the imposed K_I and