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Vlasov equilibria: Varying the temperature or the density distributions

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Abstract

Stationary selfconsistent solutions of the Vlasov–Maxwell system in a magnetized inhomogeneous plasma (so called Vlasov equilibria) provide the natural starting point for the investigation of plasma stability and of the nonlinear development of plasma instabilities in collisionless or weakly collisional regimes. In view of the different mechanisms that drive these instabilities, we discuss Vlasov equilibria with both density and temperature gradients.

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1. Introduction

Stationary solutions (equilibria) of the Vlasov–Maxwell system based on Jeans' theorem [1,2] provide a convenient starting point for the investigation of the nonlinear dynamics of electromagnetic plasmas in collisionless regimes (see Refs. [3–8]), of stellar systems such as galaxies (with the gravitational potential replacing the electromagnetic potentials, see Ref. [9] and references therein) and of selfgravitating general relativistic systems [10]. In particular, isothermal equilibria with a nonuniform density are frequently considered because, amongst other reasons, they may be expected to be more resilient to the long term dissipative effects of particle collisions. In addition, they lead to physical models that are relatively simple to solve algebraically, although such models are often affected by divergences (as, e.g., in isothermal stellar systems) or by unphysical boundary conditions. On the other hand plasma equilibria with nonuniform temperature distributions are of great interest as temperature gradients are known to affect the dynamics of magnetically confined plasmas, giving rise to new instabilities [11] or modifying important plasma processes such as magnetic reconnection [12].

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1.1. The Harris pinch

The well known Harris pinch equilibrium describes a purely magnetic (i.e., fully neutral), isothermal one-dimensional stationary plasma configuration embedded in a magnetic field of the form $\mathbf{B}(x) = B_y(x)\mathbf{e}_y + B_z\mathbf{e}_z$, with B_z constant, possibly zero. In such a configuration, the particle kinetic energy $\epsilon_j \equiv m_j\mathbf{v}^2/2$ and the z component of the canonical momentum $p_{jz} = m_jv_z + Z_jeA_z/c$ are (isolating) integrals of the particle motion. Here $j = e, i$ denotes the particle species and Z_j, m_j are the particle charge number and mass. Thus any distribution function of the form $f_j(x, \mathbf{v}) = F_j(\epsilon_j, p_{jz})$ that satisfies the appropriate positivity and integrability conditions is a stationary solution of Vlasov's equation. The set of the Vlasov–Maxwell equations is then closed by calculating the current density

$$J_z(x) = J_z(A_z(x)) \equiv \sum_j \left[Z_je\int \mathrm{d}^3v v_z F_j(\epsilon_j, p_z)\right], \tag{1}$$

along z and by solving Ampère's equation $\nabla^2 A_z(x) = -4\pi J_z(A_z(x))/c$, for the unknown vector potential, after imposing that the particle densities

$$n_j = \int \mathrm{d}^3v F_j\left(\mathbf{v}^2, p_z\right), \tag{2}$$

be equal, i.e., that the configuration is charge neutral. This allows us to find the spatial dependence of the magnetic field selfconsistently. The Harris solution [3] is obtained by choosing two isothermal distribution functions of the form

$$F_j(\epsilon_j, p_{jz}) = \frac{n_{0j}}{(2\pi T_j/m_j)^{3/2}} \exp\left[\left(-\epsilon_j + u_j^* p_{jz} - m_j u_j^{*2}/2\right)\Big/ T_j\right], \tag{3}$$

where n_{0j} is a reference density and u_j^* is the stream velocity parameter. The neutrality condition requires that the combinations $|Z_j|n_{0j}$ and $Z_ju_j^*/T_j$ be equal for electron and ions. For the (diamagnetic) current density we obtain:

$$J_z(A_z(x)) = \sum_j Z_jeu_j^*n_{0j} \exp\left[\left(u_j^*Z_jeA_z/c\right)\Big/ T_j\right]. \tag{4}$$

The distributions (3) can be written as the product of shifted Maxwellians (centered at u_j^*) times a common density distribution. The particle and the current densities have the same space dependence and the sign of the current density is the same through out the configuration. An additional uniform Maxwellian distribution can be added without changing the current distribution. By solving Ampère's equation we obtain:

$$A_z(x) = -\ln[\cosh(x)], \quad B_y(x) = -\tanh(x), \quad n(x) = 1/\cosh^2(x), \tag{5}$$

where we have normalized the vector potential on $2cT_i/(Z_ieu_i^*)$, n on the maximum density and x on $cT_i/[Z_ieu_i^*(2\pi Z_in_{0i}(T_e + T_i))^{1/2}]$.

1.2. The convolution method

The Harris distribution corresponds to uniform temperatures and stream velocities where $1/T_j$ and u_j^*/T_j could be interpreted as Lagrange multipliers in the minimization of the entropy at constant energy and canonical momentum along z. Physically interesting Vlasov solutions can be found by superimposing distributions with the same "temperature", but different stream parameters according to a procedure introduced and discussed in Ref. [5]. This approach is based on defining a "stream distribution density" $\mathcal{N}_j(u_j)$ depending on a "stream variable" u_j and gives

$$F_j(\epsilon_j, p_{jz}) = \int \mathrm{d}u_j \frac{\mathcal{N}_j(u_j)}{(2\pi T_j/m_j)^{3/2}} \exp\left[\left(-\epsilon_j + u_jp_{jz} - m_ju_j^2/2\right)\Big/ T_j\right], \tag{6}$$

which reduces to Eq. (3) for $\mathcal{N}_j(u_j) = \delta(u_j - u_j^*)n_{0j}$. Charge neutrality requires that the electron and ion stream distributions are related by $T_e\mathcal{N}_e(u_e) = T_i\mathcal{N}_i[Z_eT_iu_i/(Z_iT_e)]$. Choosing for example $\mathcal{N}_j(u_j) =$