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Asymptotic solutions and stability analysis for generalized non-homogeneous Mathieu equation

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Abstract

The asymptotic solutions and transition curves for the generalized form of the non-homogeneous Mathieu differential equation are investigated in this paper. This type of governing differential equation of motion arises from the dynamic behavior of a pendulum undergoing a butterfly-type end support motion. The strained parameter technique is used to obtain periodic asymptotic solutions. The transition curves for some special cases are presented and their corresponding periodic solutions with the periods of 2π and 4π are evaluated. The stability analyses of those transition curves in the ε–δ plane are carried out, analytically, using the multiple scales method. The numerical simulations for some typical points in the ε–δ plane are performed and the dynamic characteristics of the resulting phase plane trajectories are discussed.

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Keywords: Asymptotic solutions; Mathieu equation; Stability analysis; Transition curves; Multiple scales method; Strained parameter technique

1. Introduction

The well-known differential equation in the form of $(\ddot{u} + (\delta + 2\varepsilon\cos(2t))u = 0)$ was first introduced by Mathieu in 1868 when he determined the vibration modes of a stretched membrane having an elliptical clamped boundary. Following the work done by Mathieu, Heine in 1878 determined the first solution of the above equation using the cosine and sine series, but could not evaluate the coefficients of the series. In 1883, Floquet published his well-known theorem on the periodic solutions of linear differential equations with the periodic coefficients. The subject of infinite determinants to obtain the transition curves was subsequently introduced by Hill in 1886 [1].

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The parametric excitation has been of much of interest in many fields of engineering and physics after Mathieu's work and particularly during the last two decades. The interest arises from the many applications of the Mathieu equation in those fields. One could point out to the dynamic stability of elastic systems, amplitude distortion in the moving sound sources, electrical and thermal diffusions, electromagnetic wave guides, diffraction of sound and electromagnetic waves, accelerator dynamics, electro-hydrodynamics and parametrically-excited pendulums. Therefore, in order to cover the many areas of interest, the generalized forms of the Mathieu equation have been developed in recent years.

Mond and Cederbaum [2] analyzed the non-linear form of the Mathieu equation within the framework of the method of normal forms. They obtained the analytical conditions for the explosive instability and derived expressions for the period of vibration as well as the amplitude of the stable response.

Esmailzadeh et al. [3] found the necessary and sufficient conditions for the existence of at least one periodic solution for the Mathieu-Duffing type equation. Ng and Rand [4,5] used a combination of the averaging and perturbation methods to investigate the classical form of the Mathieu differential equation with the cubic non-linearities. Lakrad et al. [6] used the perturbation method to study the conditions of occurrence of the quasi-periodic (QP) solutions and the bursting dynamics in a self-excited quasi-periodic Mathieu oscillator. Abouhazim et al. [7] investigated the three-period oscillations in the vicinity of 2:2:1 resonance in a self-excited Mathieu equation using the perturbation method. They showed that the double reduction method can be implemented to construct three-period quasi-periodic solutions. The stability analysis of non-linear quasi-periodic solutions (with two frequencies) for some special resonance cases were reported in literatures [8–10]. Rand et al. [8] and Guennoun et al. [9] used the double multiple-scales for construction of the transition curves and Zounes and Rand [10] employed the Chirikov's overlap criterion for this purpose.

The analytical and numerical investigations into the chaotic behavior of a parametrically-exited pendulum were carried out by Bishop and Clifford [11,12]. They have identified the oscillatory orbits and analytically obtained the approximate boundaries of the escape zone. The dynamics of harmonically excited pendulum with rotational orbits was studied by Xu et al. [13]. The parameter space, bifurcation diagram, basin of attraction and the time history were used to explore the stability of rotational orbits. Younesian et al. [14] studied the non-linear generalized Mathieu equation, with cubic and quadratic non-linearities, and obtained the periodic solutions with their corresponding transition surfaces. They used the two dimensional Lindstedt–Poincare' method and showed that transition surfaces depend on the initial conditions.

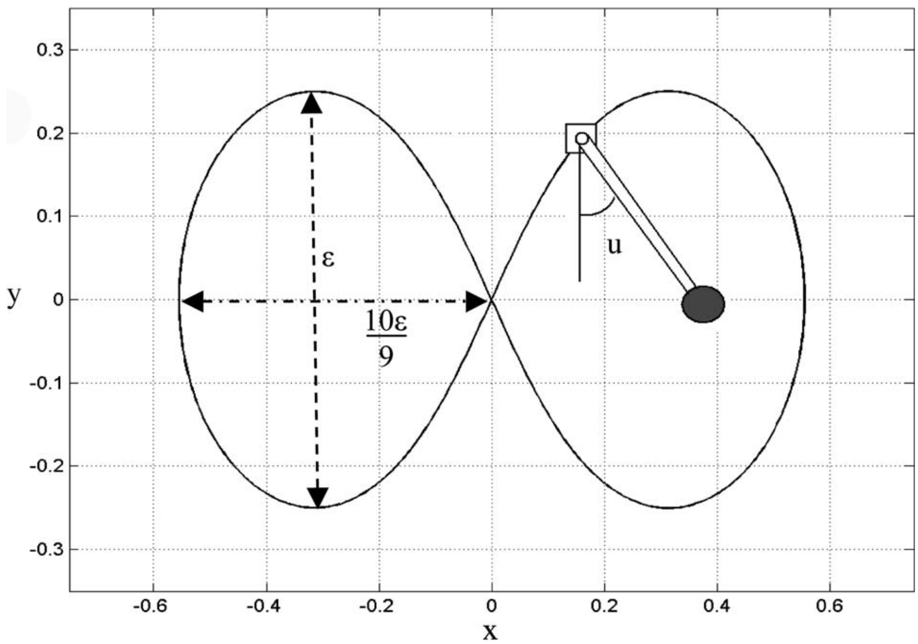

Fig. 1. A pendulum subjected to butterfly type support motion ($\varepsilon = 0.5$).