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A variety of exact wave solutions with distinct physical structures for the Boussinesq system

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Abstract

The Boussinesq system is investigated by using the tanh method and the sine–cosine method. A variety of exact travelling wave solutions with compact and noncompact structures are formally obtained. The study shows that each method gives distinct solutions and one method complements the other.

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1. Introduction

This work aims to cast light on the Boussinesq system [1,2]

$$\begin{aligned} &u_t + v_x = 0, \\ &v_t + a(u^2)_x - bu_{xxx} = 0. \end{aligned} \tag{1}$$

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This system of equations is used to model two-way propagation of certain water waves in a uniform horizontal channel filled with an irrotational and inviscid liquid [2,3].

It is well-known that the KdV equation

$$u_t + (u^2)_x + u_{xxx} = 0, \tag{2}$$

describes long nonlinear waves of small amplitude on the surface of inviscid ideal fluid. The KdV equation (2) is integrable by the inverse scattering transform and gives rise to solitons that exist due to the balance between the weak nonlinearity and dispersion of that equation. Soliton is a localized wave that has an infinite support or a localized wave with exponential tails.

The term soliton was coined by Zabusky and Kruskal [4], who performed numerical studies of the KdV equation, and found particle like waves which retained their shapes and velocities after collisions. The term soliton is coined to reflect the particle like behavior of the solitary waves under interaction.

There have been an enormous number of examples of solitons equations, verifying that the KdV is not just a freak equation [5–15]. The complexity of the nonlinear wave equations made it impossible to establish one unified method to find all solutions of these equations. Several methods, analytical and numerical, such as Backlund transformation, the inverse scattering method, bilinear transformation, the tanh method [16–18], the homogeneous balance method, and the sine–cosine ansatz, are used to treat these topics.

The $K(n,n)$ equation

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, \quad n > 1, \tag{3}$$

introduced in [13], gives rise to the so-called *compactons*: solitons with the absence of infinite wings. The delicate interaction between nonlinear convection $(u^n)_x$ with genuine nonlinear dispersion $(u^n)_{xxx}$ in the $K(n,n)$ equation (3) generates solitary waves with exact compact support that are termed *compactons*. Solitons and compactons have been receiving considerable attention in mathematical physics. For more details about solitons and compactons phenomena, the reader is advised to read the works in [1–12] for solitons, and the works in [13–15,19–32] for compactons.

The aim of the present work is to obtain travelling wave solutions of distinct physical structures for the Boussinesq system, a generalized form of the Boussinesq system, and a variant of the Boussinesq system given by

$$\begin{aligned} &u_t + v_x = 0, \\ &v_t + a(u^2)_x - bu_{xxx} = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &u_t + v_x = 0, \\ &v_t + a(u^n)_x - bu_{xxx} = 0, \end{aligned} \tag{5}$$

and

$$\begin{aligned} &u_t + v_x = 0, \\ &v_t + a(u^{-n})_x - bu_{xxx} = 0, \end{aligned} \tag{6}$$

respectively. Two strategies will be pursued to achieve our goal, namely, the tanh method [16–18] and the sine–cosine method [19–32]. The systems (4)–(6) will be used as testbed for this analysis.