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Estimating terminal velocity of rough cracks in the framework of discrete fractal fracture mechanics

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ABSTRACT

In this paper we first obtain the order of stress singularity for a dynamically propagating self-affine fractal crack. We then show that there is always an upper bound to roughness, i.e. a propagating fractal crack reaches a terminal roughness. We then study the phenomenon of reaching a terminal velocity. Assuming that propagation of a fractal crack is discrete, we predict its terminal velocity using an asymptotic energy balance argument. In particular, we show that the limiting crack speed is a material-dependent fraction of the corresponding Rayleigh wave speed.

1. Introduction

A theoretical framework for including inertial effects during a rapid crack growth was first proposed by Mott [54], who adopted the analysis of Griffith [39] as a starting point. The idea is based on a simple addition of a kinetic energy term to the expression for the total energy of the cracked system. According to Mott's extension of Griffith's criterion, the requirement that the system remains in thermodynamic equilibrium with its surroundings as the crack extends leads to the following expression in terms of the well-known fracture parameters: $G - 2\gamma = dT/da$, where G is energy release rate, γ is specific surface energy, T is kinetic energy density, and "a" is the characteristic length of the crack. Mott defined a domain R that receives stress-wave "messages" from the crack tip and then argued that the total kinetic energy can be written as $T = \frac{1}{2}\rho v^2 \int_R [(\partial u_x/\partial a)^2 + (\partial u_y/\partial a)^2] dxdy$. While Mott's analysis lacks rigour, it is instructive in the way it highlights some of the important features of a running crack without excessive mathematical complication.

The first important contribution to the problem of a moving crack with constant velocity was the work of Yoffe [89]. The Yoffe problem consists of a mode I crack of fixed length traveling through an elastic body at a constant speed under the action of uniform remote tensile loading. Yoffe [89] obtained the stress distribution near the tip of a rapidly propagating crack in a plate of isotropic elastic medium. The result was that the stresses depend on the crack tip velocity and reduce to the solution of Inglis [41] when the velocity is zero.

Roberts and Wells [70] used Mott's extension of Griffith's criterion to predict the limiting velocity of the crack extension. By taking the boundary of the region R to be a circle of radius r centered at the crack tip, they estimated $r \approx c_0 t$, where $c_0 = \sqrt{E/\rho}$ is the longitudinal sound wave speed. They defined this cutoff region as the border of the disturbed zone by the stress waves emanated from the crack tip. Using this assumption and taking a stress field similar to that of the static

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case they roughly estimated the limiting crack velocity to be about $0.38c_0$ when $\nu = 0.25$. Steverding and Lehnigk [77] studied the problem of the response of cracks to stress pulses and found an equation of motion for such cracks. They also obtained the limiting velocity of crack extension caused by stress pulses by using asymptotic solutions to be about $0.52c_R$, where c_R is the Rayleigh wave speed. There have also been some other efforts on finding the equation of motion for dynamically propagating cracks. Berry [8,9] was the first to find an equation of motion for dynamic propagation of cracks. He found out that the details of the motion of a crack are determined by the state of stress at the point of fracture, and that the observed critical stress is (infinitesimally) greater than that given by the Griffith's criterion and is probably determined by the size of the defect in the sample and the rate of straining. He also obtained solutions for the equation of motion for fracture in tension and fracture in cleavage in both constant force and constant velocity cases. See Bouchbinder et al. [15] for a recent review of dynamic fracture mechanics.

The inadequacy of the classical fracture mechanics theories in problems such as predicting infinite strength for elastic bodies without any cracks, for example, was the motivation for some researchers to propose new failure theories. Novozhilov [56] introduced a non-local stress criterion and gave the condition of the brittle crack propagation in mode I as $\sigma^* \equiv \langle \sigma_y(x) \rangle_0^{a_0} = \sigma_c$, where $\langle . \rangle_0^{a_0}$ is spatial averaging over the interval $[0, a_0]$, $\sigma_y(x)$ is the complete (not only asymptotic) stress field around the crack tip ($x = 0$), σ_c is the ideal strength of the material, and a_0 is the minimum admissible crack advance named by him a *fracture quantum*. According to Novozhilov this criterion can be used only with the complete expression of the stress field, and not with its asymptotic form. However, the complete expression is rarely known. Another restriction in Novozhilov's approach was that the size of fracture quantum assumed to be the atomic spacing. Pugno and Ruoff [60] introduced their so-called quantized fracture mechanics (QFM) approach, which modified Novozhilov's theory. In QFM, the restrictions of Novozhilov's theory were removed and this made QFM a useful approach for analysis of very short cracks (see also Krasovs'kyi [43], Morozov and Petrov [52], Cornetti et al. [21], Leguillon [46] for more related works). In their approach the differentials in Griffith's criterion were replaced by finite differences (see Wnuk and Yavari [81] for a discussion). For vanishing crack length, QFM predicts a finite ideal strength in agreement with the prediction of Orowan [57].

In most models in fracture mechanics cracks are assumed to be smooth for mathematical convenience. However, in reality fracture surfaces are rough and "roughness" evolves in the process of crack propagation. Fracture surfaces of many materials of interest are fractals, a fact that has been experimentally established by many researchers. A fractal dimension (roughness exponent) is not enough to uniquely specify a fractal set and this is why all one can hope for achieving having only a fractal dimension (roughness exponent) is a qualitative analysis. Effects of fractality on fracture characteristics of rough cracks have been investigated by several groups in the past two decades (see Mosolov [53], Gol'dshteĭn and Mosolov [37], Gol'dshteĭn and Mosolov [38], Balankin [6], Borodich [11], Carpinteri [17], Cherepanov et al. [20], Xie [83], Xie [84], Yavari et al. [85–88], Wnuk and Yavari [79–82], and references therein). Here our interest is to estimate the observed terminal velocity of a rough crack propagating dynamically in an elastic medium.

Wnuk and Yavari [81] extended quantized (finite) fracture mechanics ideas for fractal cracks. They presented a modification of the classical theory of brittle fracture of solids by relating discrete nature of crack propagation to the fractal geometry of the crack. Their work is based on the idea of using an equivalent smooth blunt crack with a finite radius of curvature at its tips for a given fractal crack [79,80]. By taking the radius of curvature of the equivalent blunt crack as a material property, they showed that fractal dimension of the crack trajectory is a monotonically increasing function of the nominal crack length. This result was an analytical demonstration of the mirror–mist–hackle phenomenon for rough cracks. Later they showed that assuming a cohesive zone ahead of a fractal crack, the size of the cohesive zone increases while the crack propagates [82].

To our best knowledge, the only contributions related to the dynamic fracture of fractal cracks are Xie [84] and Alves [3]. Xie [84] introduced a fractal kinking model of the crack extension path to describe irregular crack growth. Then by using the formula proposed by Freund [35] for calculating dynamic stress intensity factor for arbitrary crack tip motion he calculated the stress intensity factor for the assumed fractal crack path. He concluded that the reason for having terminal velocities lower than the Rayleigh wave speed is the fractality of the crack path. Alves [3] used a fractal model for rough crack surfaces in brittle materials. He tried to explain the effect of fractality of fracture surfaces on the stable (quasi-static) and unstable (dynamic) fracture resistance. He concluded that fractal dimension has a strong influence on the rising of the R-curve in brittle materials. He also argued that the reason for having terminal velocities lower than Rayleigh wave speed is the roughness of the fracture surfaces that makes the nominal (projected) and local crack tip velocities different.

Dulaney and Brace [27] modified Mott's analysis of energy balance and showed that for a crack of initial and current lengths c_0 and c, respectively, $\dot{c} = v_L(1 - c_0/c)$ compared to the similar equation $\dot{c} = v_L(1 - c_0/c)^{\frac{1}{2}}$ obtained by Mott. Here v_L is the terminal velocity and is proportional to $\sqrt{E/\rho}$. They also carried out tests of terminal velocity on PMMA specimens and showed that the measured velocity differs only about 10% from the predicted value by Roberts and Wells [70]. Recently, Chekunaev and Kaplan [18,19] studied the terminal velocity by replacing a mode I crack under pressure on its faces by considering cohesive forces and replacing the crack by an equivalent distribution of dislocations. They obtained simple expressions for the potential and kinetic energies of the environment of the moving crack. They also obtained an expression for an equivalent mass for the crack tip, i.e. a point mass that has the same kinetic energy as the whole cracked system. Their equivalent mass depends on a truncation radius R in the form of $\ln(2R/a)$, where a is the half crack length. For a uniform external pressure p_0 they showed that the crack tip speed can be expressed as $v = v_L(1 - a_{cr}/a)$, where a_{cr} depends on both mechanical properties and p_0. Their terminal velocity has the form $v_L = g(\nu, R/a)\sqrt{E/\rho}$ that they approximate by $v_L = \hat{g}(\nu)\sqrt{E/\rho}$, for