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Weight function approach for determining crack extension resistance based on the cohesive stress distribution in concrete

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ABSTRACT

The use of universal form of weight functions for determining the K_R-curves associated with the cohesive stress distribution for complete fracture process of three-point bending notched concrete beam is reported in the paper. Closed form expressions for the cohesion toughness with linear and bilinear distribution of cohesive stress in the fictitious fracture zone during monotonic loading of structures are obtained. Comparison with existing analytical method shows that the weight function method yields results without any appreciable error with improved computational efficiency. The stability analysis and the size-effect study using K_R-curves of crack propagation are also described.

1. Introduction

In heterogeneous aggregative materials such as cement mortar, concrete and fiber-reinforced concrete; numerous new interfaces and voids are created that lead to several additional energy absorption mechanisms. During fracture process of concrete composites, a large region of pseudo-plasticity termed as fracture process zone or nonlinear process zone or slow crack growth develops at the tip of a propagating crack, which requires additional energy input from the loading system. In 1960s, many researchers determined the fracture toughness generally expressed by critical stress intensity factor (K_{IC}) or critical energy release rate (G_{IC}) from experiments using linear elastic fracture mechanics (LEFM). Subsequently, it was observed that the fracture toughness for the same mix of concrete so determined mainly varied with the test specimen dimensions, length of crack extension and geometry. It is now accepted that the concept of linear elastic fracture mechanics is inapplicable to normal size structural members made of cementitious materials and therefore, the study of fracture process and crack propagation of concrete needs a special treatment using nonlinear fracture mechanics. Fictitious crack model originally introduced by Hillerborg et al. [1] and crack band model initially proposed by Bazant and Oh [2] based on principles of nonlinear fracture mechanics are widely used in practice for realistic predictions with a wide range of concrete fracture parameters. However, for simple engineering problems; the use of nonlinear fracture mechanics for prediction of fracture behavior requires the cumbersome finite element analyses. This necessitates one to wonder for approximate analytical method using linear elastic fracture mechanics concept which can deal with the nonlinear process zone of concrete fracture. For *quassibrittle* materials such as concrete, one of such methods is known as *R*-curve (crack growth resistance curve) concept which is attributed to extension of equivalent elastic crack. The equivalent elastic crack is a traction free crack in elastic medium which yields the same compliance as the actual crack associated with a large nonlinear fracture process zone.

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Nomenclature

a_o	initial crack length
a_c	effective crack length at peak (critical) load
b	width of beam
c_1, c_2	material constants for nonlinear softening function
D	depth of beam
Δa_c	effective crack extension at and beyond the critical load
E	modulus of elasticity of concrete
$F(x/a, a/D)$	the standard Tada Green's function for edge cracks subjected to pair of forces normal to the crack face
f_t	uniaxial tensile strength of concrete
H	thickness of clip gauge holder
K_I	stress intensity factor in Mode I fracture
K_{IC}^{ini}	the initiation toughness
K_{IC}^{un}	the unstable toughness
K_I^{COH}	the cohesive toughness
K_C^t	total crack extension resistance
$m(x,a)$	weight function
M_1, M_2, M_3, M_4	parameters of weight function
r_u	undamaged length of the ligament during crack propagation
S	span of beam
P_{ini}	crack initiation load
P_u	maximum applied load
σ	cohesive stress corresponding to crack opening displacement w
$\sigma_s(CTOD_c)$	cohesive stress at critical value of crack-tip opening displacement ($CTOD_c$)
$\sigma(w_t)$	cohesive stress at the initial notch-tip
w	crack opening displacement ahead of the crack-tip
w_c	critical value of crack opening displacement at which the value $\sigma = 0$
w_t	crack opening displacement at the initial notch-tip

The *R*-curve is represented by a plot between the crack extension resistance expressed in terms of either strain energy release rate *G* or stress intensity factor *K* and the corresponding crack extension Δa. It is generally called as G_R-curve and K_R-curve depending upon the unit of the crack resistance parameters strain energy release rate and stress intensity factor, respectively. The shape of the *R*-curve determines the onset of crack instability. For most of the engineering materials except truly brittle material, a rising *R*-curve is observed due to slow stable crack extension. As one crack extends, a rising *R*-curve produces several small cracks because the required stress intensity to propagate the crack will increase to a point where it is favorable to propagate a different crack. On the other hand, a flat *R*-curve for ideal brittle materials will give rise to a single and possibly catastrophic crack. Crack extension dependence of the crack growth resistance curve behavior is one of the important subjects in the study of fracture mechanics of concrete. In past, many studies have been performed with the K_R-curves on concrete fracture by different groups of researchers: Wecharatana and Shah [3,4], Bazant and Cedolin [5], Mai [6], Hilsdorf and Brameshuber [7], Karihaloo [8], Bazant et al. [9], Bazant and Kazemi [10,11], Bazant and Jirasek [12], Elices and Planas [13], Planas et al. [14], Xu and Reinhardt [15,16] and Reinhardt and Xu [17].

Recently, Xu and Reinhardt [15,16] and Reinhardt and Xu [17] have introduced K_R-curve method for complete fracture process description of concrete. Their approach differs from the conventional method originally proposed by Irwin [18] and Kraft et al. [19] in early 1960s. The distribution of cohesive stress along the fictitious fracture zone at different stages of loading conditions are taken into account in order to evaluate the K_R-curve for three-point bending test of concrete beam [15,16]. Double-*K* fracture parameters; the initiation toughness K_{IC}^{ini} and the unstable toughness K_{IC}^{un} are introduced in the K_R-curve analysis. The initiation toughness is defined as the inherent toughness of the materials, which holds for loading at crack initiation when material behaves elastically and micro cracking is concentrated to a small-scale in the absence of main crack growth. The value K_{IC}^{ini} is evaluated using linear elastic fracture mechanics relations for the crack initiation load P_{ini} and the initial notch length a_o of the cracked structures. Similar to the nonlinear fracture models [1,2] attributed to numerical approach, the distribution of cohesive stress is also considered in the K_R-curve analysis and double-*K* fracture criterion. The cohesive stress starts acting in the fictitious fracture zone as the external load on the structure increases beyond the value of P_{ini}. The contribution of toughness due to cohesive stress is termed as cohesive toughness of the material. During the crack propagation, the contribution of cohesive toughness increases with the increase in process zone length. The total toughness at the critical condition is known as unstable toughness K_{IC}^{un} which is regarded as one of the material fracture parameters at the onset of the unstable crack propagation. The cohesive toughness depends upon cohesive stress distribu-