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Calibration of a higher-order continuum model using global and local data

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ABSTRACT

Part of the reliability of numerical models for the description of localized failure in quasi-brittle materials strongly depends on the correct identification of the model parameters. Particularly, the solution of an inverse problem is required for those model parameters for which a clear physical meaning is not established and, as a consequence, direct measurements in laboratory tests are not possible. This is the case for the continuum gradient damage model, regularized by the introduction of a length scale parameter. For this model, an inverse strategy for the identification of parameters is proposed. Local experimental data related to the width of the fractured area of the specimen are used, in addition to global force–deformation curves, so that the inverse problem results to be well posed. Two series of experimental data, concerning different specimen sizes and loading conditions, are considered in order to investigate the limits and potential of the calibrated model in terms of predictive capabilities.

1. Introduction

The significant evolution of computational tools in the last years makes practicable nonlinear analysis of concrete structures as a modern way for optimizing design in a better possible way. Within this framework, the constitutive modelling of the tensile behaviour of concrete is fundamental in order to numerically simulate the strain localization process responsible for macroscopic fracture phenomena. For this purpose, different computational models have been developed following either the discrete modelling or the continuum modelling approach. However, assuming that these models are capable of correctly describing the fracture process according to a qualitative point of view, their reliability, in reproducing quantitatively correct results, significantly relies on the identification of the constants characterizing the model equations (model parameters). Some of them can be directly measured during laboratory tests, but for those model parameters defined at the material point level and for which a real physical interpretation is not clearly established, the solution of an inverse problem is required [1]. Hence, the estimated parameter is obtained by minimizing, iteratively, the discrepancy between experimental and computational data, so that innovative computational and experimental techniques may be coupled to extract intrinsic material properties from measured structural responses. This is the case, for instance, for the length scale parameter, introduced in the nonlocal continuum damage models in order to regularize the local approach. Regarding this, interesting investigation can be found, for instance, in [2–5]. However, more extensive research is required concerning the assessment of the well-posedness of the identification problem (also influenced by the choice of experimental data, in terms of quality and quantity, involved in the solution), the choice of the adopted inverse techniques (as suitable searching tools in the parameters space in terms of effectiveness, efficiency and robustness) and the assessment of the calibrated numerical

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List of symbols

$\mathbf{b}$	body forces
c	gradient parameter ($= l^2/2$)
$d^t_{\mathrm{comp}}(\mathbf{x})$	computational FPZ width at time t
d^t_{exp}	experimental FPZ width at time t
$\mathbf{d}_{\mathrm{comp}}(\mathbf{x})$	computational vector of FPZ widths during fracture process
$\mathbf{d}_{\mathrm{exp}}$	experimental vector of FPZ widths during fracture process
$\mathbf{e}^t(\mathbf{x})$	error between experimental and computational data ($= \mathbf{y}^t_{\mathrm{exp}} - \mathbf{y}^t_{\mathrm{comp}}(\mathbf{x})$)
f	loading function
$f(\mathbf{x})$	objective function in the inverse problem
f_{cc}	compressive strength
f_{ct}	tensile strength
$\mathbf{h}_t(\mathbf{x})$	forward operator at time t
l	length scale parameter
n_y	$\mathbf{y}^t_{\mathrm{exp}}$ or $\mathbf{y}^t_{\mathrm{comp}}$ vector dimension
n_x	$\mathbf{x}$ vector dimension
$\mathbf{n}$	outward unit normal to Ω
p_1	weighting coefficient for the global data in $f(\mathbf{x})$
p_2	weighting coefficient for the local data in $f(\mathbf{x})$
t	time, step
$\hat{\mathbf{t}}$	prescribed tractions
$\mathbf{u}$	displacement field
$\hat{\mathbf{u}}$	prescribed displacements
$\mathbf{x}$	model parameters vector
$\hat{\mathbf{x}}_t$	mean value estimate of the model parameters vector at time t
$\hat{\mathbf{x}}_0$	initial guess of $\hat{\mathbf{x}}_t$
$\mathbf{y}_{\mathrm{comp}}(\mathbf{x})$	computational data in a batch form
$\mathbf{y}^t_{\mathrm{comp}}(\mathbf{x})$	computational data at time t
$\mathbf{y}_{\mathrm{exp}}$	measured experimental data in a batch form
$\mathbf{y}^t_{\mathrm{exp}}$	measured experimental data at time t
$\mathbf{y}^*_{\mathrm{exp}}$	measurable response of the real system in a batch form
$\mathbf{y}^{t*}_{\mathrm{exp}}$	measurable response of the real system at time t
$\mathbf{C}^t_{\mathrm{exp}}$	experimental data covariance matrix at time t associated with $\mathbf{y}^t_{\mathrm{exp}}$
$\hat{\mathbf{C}}_t$	model parameters covariance matrix at time t associated with $\hat{\mathbf{x}}_t$
$\hat{\mathbf{C}}_0$	initial model parameters guess covariance matrix associated with $\hat{\mathbf{x}}_0$
CMOD	crack mouth opening displacement of the specimen
$\mathbf{D}^{\mathrm{el}}$	fourth-order linear elastic constitutive tensor linear elastic constitutive tangent matrix
E	Young's modulus
$F^t_{\mathrm{comp}}(\mathbf{x})$	computational force applied to the specimen at time t
F^t_{exp}	experimental force applied to the specimen at time t
I'_1	first invariant of the strain tensor
J'_2	second invariant of the deviatoric strain tensor
$\mathbf{K}_t$	gain matrix at time t
$\mathbf{L}$	differential operator
$\mathbf{N}$	outward unit normal to Ω components matrix
$\mathbf{S}_t$	sensitivity matrix at time t

Greek symbols

α	softening parameter in damage evolution law
β	softening parameter in damage evolution law
$\varepsilon_{\mathrm{eq}}$	local equivalent strain
$\bar{\varepsilon}_{\mathrm{eq}}$	nonlocal equivalent strain
$\boldsymbol{\varepsilon}$	strain tensor strain array in engineering notation
η	$f_{\mathrm{cc}}/f_{\mathrm{ct}}$
κ	deformation history variable
κ_i	strain threshold of damage initiation
$\kappa_{\max}$	maximum value of equivalent strain
ν	Poisson's ratio
$\boldsymbol{\sigma}$	stress tensor stress array in engineering notation
ω	scalar damage variable