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A simple volume-of-fluid reconstruction method for three-dimensional two-phase flows

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ABSTRACT

A new PLIC (piecewise linear interface calculation)-type VOF (volume of fluid) method, called APPLIC (approximated PLIC) method, is presented. Although the PLIC method is one of the most accurate VOF methods, the three-dimensional algorithm is complex. Accordingly, it is hard to develop and maintain the computational code. The APPLIC method reduces the complexity using simple approximation formulae. Three numerical tests were performed to compare the accuracy of the SVOF (simplified volume of fluid), VOF/WLIC (weighed line interface calculation), THINC/SW (tangent of hyperbola for interface capturing/slope weighting), THINC/WLIC, PLIC, and APPLIC methods. The results of the tests show that the APPLIC results are as accurate as the PLIC results and are more accurate than the SVOF, VOF/WLIC, THINC/SW, and THINC/WLIC results. It was demonstrated that the APPLIC method is more computationally efficient than the PLIC method.

1. Introduction

Two-phase flows are essential in many research fields; for example, in relation to cloud and precipitation droplets in the atmosphere, water waves, cooling devices, oil and gas pipelines, chemical industrial plants, and thermal power stations. In the recent decades, many interface tracking methods for simulating two-phase flows have been developed. The VOF (volume of fluid) method, originated by Hirt and Nichols [1], is one of the most widely used algorithms. Excellent reviews of the VOF method have been given by Rudman [2], Rider and Kothe [3], Scardovelli and Zaleski [4], and Pilliod and Puckett [5].

The VOF method is based on the spatial discretization of a characteristic function to distinguish between two phases, and the reconstruction of the interfaces for advection. Suppose that we wish to simulate an incompressible two-phase ('light' and 'dark') flow in the three-dimensional Cartesian space $\boldsymbol{x} = (x_1, x_2, x_3)$. The characteristic function for the flow is defined as

$$\chi(\boldsymbol{x}) = \begin{cases} 0 & \text{if there is light fluid at point } \boldsymbol{x}, \\ 1 & \text{if there is dark fluid at point } \boldsymbol{x}. \end{cases} \tag{1}$$

The interfaces between the two phases are represented by the discontinuity of the characteristic function. In this paper, we suppose that a computational grid composed of cubic cells of a edge Δx is used. Extension of our analysis to general regular grids is straightforward. By descretizing the characteristic function in a computational cell (i, j, k), we can obtain the volume fraction

$$C_{i,j,k} = \frac{1}{(\Delta x)^3} \int_{\Omega_{i,j,k}} \chi(\boldsymbol{x})\, d\boldsymbol{x}, \tag{2}$$

where $\Omega_{i,j,k}$ is the domain of the cell. It is obvious from the definition that

$$C_{i,j,k} \begin{cases} = 0 & \text{if the cell is filled by light fluid}, \\ \in (0,1) & \text{if the cell contains both fluids (interface cell)}, \\ = 1 & \text{if the cell is filled by dark fluid}. \end{cases} \tag{3}$$

The VOF method reconstructs the shape of the interface in each interface cell to evaluate VOF advection fluxes. Various schemes for VOF reconstruction have been presented. The PLIC (piecewise linear interface calculation) method [6,7] reconstructs an interface in a cell as a plane (in three-dimensional space) or a line (in two-dimensional space) with a given normal vector. The SLIC (simple line interface calculation) method [8] assumes the shape of an interface to be a plane parallel to one of the cell faces. The VOF/WLIC (weighted line interface calculation) method [9] evaluates an advection flux through a cell face as a weighted sum of SLIC fluxes. The SVOF (simplified volume of fluid) method [10] is similar to the VOF/WLIC method, except for the weight formula. In the THINC (tangent of hyperbola for interface capturing) method [11], interfaces are represented by the use of the hyperbolic tangent. Improved THINC methods have also been proposed [9,12,13].

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Although it is known to be one of the most accurate reconstruction methods, a three-dimensional implementation of the PLIC method is a troublesome task. The PLIC method requires the solution of two geometric problems, as to a cut-volume of a cube by a plane, which are very complicated especially in three-dimensional cases. Scardovelli and Zaleski have provided two sophisticated algorithms (hereafter called the SZ algorithms) to solve these problems [14]. Although the SZ algorithms make the implementation of the PLIC method easier because of their compactness, these are still too complex for quick and easy implementation. Computational routines that implement the SZ algorithms must involve multiple "if" statements, which make it hard to develop and maintain the routines, and potentially inhibit its optimal compilation, especially for processors susceptible to conditional branches, e.g., deeply pipelined processors, processors with SIMD (single instruction multiple data) operations, vector processors, and GPUs (graphics processing units) [15].

In this paper, a PLIC-type VOF method called the APPLIC (approximated PLIC) method is presented. In the APPLIC method, interfaces are reconstructed in a similar manner as in the PLIC method, except that the geometric problems are solved through the use of simple approximation formulae.

This paper is organized as follows. In Section 2, we describe the APPLIC method. Section 3 compares the accuracy and computational efficiency of the APPLIC method with some existing VOF methods. Finally, conclusions are summarized in Section 4.

The following vector notation is used throughout this paper. Bold letters denote three-dimensional vectors and the corresponding non-bold letters with subscripts 1, 2, or 3 denote the vector components. For example, $\boldsymbol{u} = (u_1, u_2, u_3)$ and $\boldsymbol{m}''_{\mathrm{A}} = (m''_{\mathrm{A},1}, m''_{\mathrm{A},2}, m''_{\mathrm{A},3})$. We use $\|\boldsymbol{m}\|_n$ to represent the n-norm of $\boldsymbol{m}$; namely, $\|\boldsymbol{m}\|_1 = |m_1| + |m_2| + |m_3|$ and $\|\boldsymbol{m}\|_2 = (m_1^2 + m_2^2 + m_3^2)^{1/2}$. The expression $\boldsymbol{m} \geq a$ stands for the condition $m_1 \geq a$, $m_2 \geq a$, and $m_3 \geq a$.

2. Method

2.1. The PLIC method using the SZ algorithms

In this paper, we use directional splitting for advection and a regular staggered grid where velocity components u_1, u_2, and u_3 are stored at the centers of the cell faces $\{(i+1/2, j, k)\}$, $\{(i, j+1/2, k)\}$, and $\{(i, j, k+1/2)\}$, respectively. We assume that the Courant–Friedrichs–Lewy (CFL) condition,

$$\frac{|u_l|\Delta t}{\Delta x} < 1 \quad \text{for all } l \in \{1, 2, 3\}, \tag{4}$$

holds, where Δt is the time step size.

Let ϕ be the face, and u_I ($I = 1, 2$, or 3) the velocity component placed on ϕ. Let Ω be the donor cell, which is the cell that has ϕ as a cell face and lies on the upwind side of u_I. Let ϕ^* be the opposite face of the face ϕ in the cell Ω. The cell Ω is partitioned into two subcells by the section σ parallel to ϕ and laid $|u_I|\Delta t$ away from ϕ. Let Ω_{A} and Ω_{B} be the subcells of Ω between ϕ and σ and between σ and ϕ^*, respectively (see Fig. 1). The section σ is always located between ϕ and ϕ^* because of the CFL condition. Let C_{A} and C_{B} be the partial volume fractions in Ω_{A} and Ω_{B}, respectively, defined as

$$C_{\mathrm{A}} = \frac{1}{(\Delta x)^3} \int_{\Omega_{\mathrm{A}}} \chi(\boldsymbol{x})\, d\boldsymbol{x}, \tag{5}$$

$$C_{\mathrm{B}} = \frac{1}{(\Delta x)^3} \int_{\Omega_{\mathrm{B}}} \chi(\boldsymbol{x})\, d\boldsymbol{x}. \tag{6}$$

It is obvious that $C_{\mathrm{A}} \in [0, 1]$ and $C_{\mathrm{B}} \in [0, 1]$. From the definitions, we have

$$C_{\mathrm{A}} + C_{\mathrm{B}} = C, \tag{7}$$

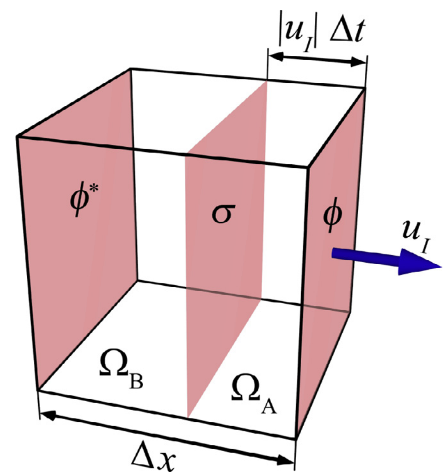

Fig. 1. Schematic illustration of a donor cell with respect to a flux through a cell face ϕ. The cell is divided into two subcells, Ω_{A} and Ω_{B}, by the section σ parallel to ϕ and laid $|u_I|\Delta t$ away from ϕ.

where C is the volume fraction in the donor cell Ω.

The computational advection flux of the volume fraction through the face ϕ (i.e., the amount of the volume fraction through the face during Δt) is obtained via

$$F = C_{\mathrm{A}}\, \mathrm{sgn}\, u_I, \tag{8}$$

where sgn is the sign function defined as

$$\mathrm{sgn}\, x = \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{if } x < 0. \end{cases} \tag{9}$$

In some cases, C_{A} is easily determined by

$$C_{\mathrm{A}} = \begin{cases} 0 & \text{if } C = 0 \text{ or } |g| = 0, \\ |g| & \text{if } C = 1, \end{cases} \tag{10}$$

where $|g|$ denote the local CFL number in the cell Ω with respect to the flux through the face ϕ:

$$g = \frac{u_I \Delta t}{\Delta x}. \tag{11}$$

Because of the CFL condition, g must be in the range $(-1, 1)$. If $C \in (0, 1)$ and $|g| > 0$, C_{A} is determined through the reconstruction of the interface in the donor cell Ω.

Here, we define the two geometric problems crucial to the PLIC method, which are mutually inverse. Consider a unit cube $U = \{\boldsymbol{x} \in [0, 1]^3\}$ and an oriented plane $P(\alpha, \boldsymbol{m}) = \{\boldsymbol{x} | \boldsymbol{m} \cdot \boldsymbol{x} < \alpha\}$, where $\boldsymbol{m}$ is the normal vector of the plane, and α is the plane constant. Note that an oriented plane is not a thin object without volume, but is a solid object with an inside and an outside. Let V be the volume of the intersection between the unit cube and the oriented plane. One of the problems, called the forward problem, is to determine the value of V for given α and $\boldsymbol{m}$. The other problem, called the inverse problem, is to determine the value of α for given V and $\boldsymbol{m}$. Namely,

$$V(\alpha, \boldsymbol{m}) = \int_{U \cap P(\alpha, \boldsymbol{m})} d\boldsymbol{x}, \tag{12}$$

$$\alpha(V', \boldsymbol{m}) = \alpha' \quad \text{such that} \quad V(\alpha', \boldsymbol{m}) = V'. \tag{13}$$

To reduce the complexity of the problems, the SZ algorithms restrict $\boldsymbol{m}$ to a vector so that $\boldsymbol{m} \geq 0$ and $\|\boldsymbol{m}\|_1 = 1$.

The functions $V(\alpha, \boldsymbol{m})$ and $\alpha(V, \boldsymbol{m})$ have the following properties [4].