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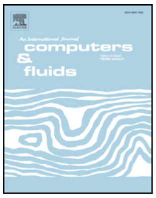

Swimming performance and vorticity structures of a mother–calf pair of fish

Fang-Bao Tian [a,*], Wenquan Wang [a,b], Jian Wu [c], Yi Sui [d]

[a] School of Engineering and Information Technology, University of New South Wales, Canberra, ACT 2600, Australia
[b] Department of Engineering Mechanics, Kunming University of Science and Technology, Kunming ,Yunnan 650051, PR China
[c] Institut PPRIME, Département Fluide-Thermique-Combustion Boulevard Pierre et Marie Curie, BP 30179 86962 Futuroscope-Chasseneuil, France
[d] School of Engineering and Materials Science, Queen Mary University of London, London E1 4NS, United Kingdom

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ABSTRACT

Flow around two foils of different sizes is investigated by solving the incompressible Navier–Stokes equations numerically with a space-time finite element method to study the swimming performance and vorticity structures of a mother–calf pair of fish. Based on simulations over a wide range of the relative position, Reynolds number, phase difference and phase velocity between the foils, we discuss in detail the drag coefficient, hydrodynamic power, root-mean-square value of the lift, and vorticity structures. It is found that both the mother and calf fish benefit in tandem and staggered arrangements. By changing their phase difference and phase velocity, the fish can get better performance in terms of thrust increase and hydrodynamic power saving. The mechanism of these strategies which are used to improve the performance is enhancing a reverse Kármán street or diminishing a Kármán street.

1. Introduction

The vorticity control for efficient propulsion is a central issue for aerial and aquatic animals. The study of this issue sheds light on the understanding of design and control concepts of aquatic and aerial animals. Owing to its fundamental importance and potential applications, a great effort has been made in the past half century to study this issue. Some reviews of this subject can be found in Refs. [12,18,41,61,74].

A common strategy for animals to locomote through a fluid is to use the bodies and/or the appendages (e.g. fins and wings) to make waving, flapping and pitching motions [3,10,19,30,39,64]. Based on the observations and measurements of animal locomotion, theoretical analyses have been performed to study biofluid dynamics of the animal swimming and flying. For example, Taylor [43] developed the resistive force theory to predict the hydrodynamic force of the swimming microscopic organism at low Reynolds numbers where the inertial force can be neglected. Later, this method was extended to study the narrow and long animals swimming at finite Reynolds numbers and the microscopic organisms swimming with spiral traveling wave propulsion [44,45]. The elongated body theory first proposed by Jones [24] was extended by Lighthill [27,28] to address the undulatory propulsion of small amplitude at high Reynolds numbers where the viscous shear force is ignorable. Then Lighthill [29] further applied this method to study the finite-amplitude swimming. Later, the elongated body theory has been applied to the slender fish with fins and the three-dimensional cases with finite-amplitude motions [8,31,73]. Wu [72] proposed the two-dimensional waving plate theory to study the swimming of a waving plate. Cheng et al. [9] extended the two-dimensional waving plate theory to study the three-dimensional waving plates. From the above discussions, we notice that the theoretical analyses were proposed typically for simplified geometries at low or high Reynolds numbers [11,30,43]. However, numerous animals are geometrically complicated, and swim or fly in the intermediate Reynolds number regime. The simple theories introduced above will fail in studying the locomotion of animals with complex shapes and kinematics appropriate to this Reynolds number range. Instead, these challenges can be naturally overcome by using the direct numerical approach, thus it is highly desirable to employ such a technique to reveal the relevant mechanisms of locomotion based on some typical models.

Numerical simulations have been extensively developed to study a viscous flow past traveling wavy bodies since 1990s. Specifically, Videv and Doi [63] presented a study of a uniform flow around a rigid two-dimensional NACA0012 foil performing pitching oscillations with a numerical method that solved the two-dimensional laminar incompressible Navier–Stokes equations in vorticity-stream function formulation defined in moving noninertial frame of

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[*] Correspondence author. Tel.: +61 2 626 88533.
E-mail address: f.tian@adfa.edu.au, fangbao.tian@unsw.edu.au, onetfbao@gmail.com (F.-B. Tian).

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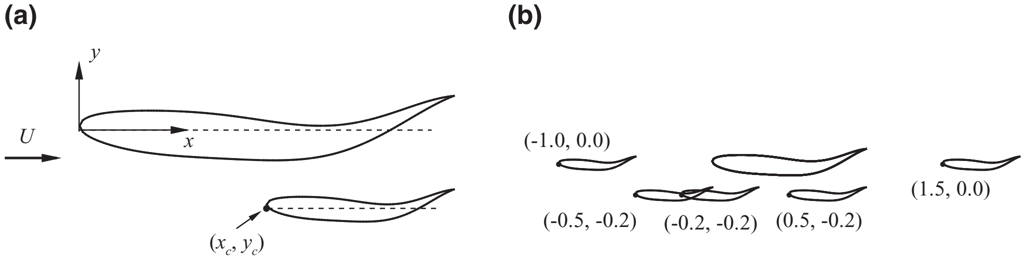

Fig. 1. (a) Schematic description of a mother–calf pair of fish which are modeled by two traveling wavy foils. In this work, the NACA0012 profile is used; and (b) five typical positions of the calf fish relative to the mother fish.

reference. The deforming-spatial-domain/stabilized space-time method, first given by Mittal and Tezduyar [35], has been applied to study flapping wing propulsion [22]. Liu et al. [32,33] numerically analyzed the hydrodynamics of an undulatory swimming tadpole. A direct numerical simulation was performed by Shen et al. [40] to study the turbulent flow over a flexible wall undergoing a streamwise traveling wave motion and further to reveal the hydrodynamic behaviors based on drag reduction and optimal propulsive efficiency. The authors found that as c/U (c is the phase velocity of the traveling wave and U is the free stream velocity) increases, the pressure drag decreases monotonically, becoming negative (thrust) at $c/U \sim 1$. In addition, the net power required reaches its minimum value at $c/U \approx 1.2$, which is the value cited for traveling wave-like thunniform swimming motion of live fish in nature [40,62]. Dong and Lu [14] and Wu et al. [70] investigated the hydrodynamics and flow structures around a traveling wavy body to gain an understanding of fish-like swimming mechanisms. Their results are consistent with those predicted by Shen et al. [40]. In addition, the authors found that effective propulsive motion for the traveling wavy plate with the amplitude around 0.1 is acquired with suitable power consumption and the traveling wavy plate with increasing amplitude is of benefit to generating effective performance of propulsion. A body with a traveling wave surface was investigated by Tian et al. [52,57] to understand the mechanisms of this propulsive strategy. It was found that the traveling wave surface propulsion is an efficient and quiet propulsion strategy. The characteristics of flow over traveling wavy foils in a side-by-side arrangement and a tandem arrangement were studied by Dong and Lu [15] and Deng et al. [13]. It was found that the fish may enjoy benefit from the vortex interaction. A filament has been taken as a simple model of swimming animal to study the hydrodynamic interactions of structure-fluid and multiple flexible bodies [2,20,21,25,49,53–55,71,75–77]. It was found that the passive filament can get drag reduction or even thrust from the vortex-filament or multi-filament interaction.

One of the important issues in fish swimming is the vorticity control [18,59,78]. The propulsive performance of an aquatic animal is correlated with the efficient generation of vorticity wake, which is often controlled by dimensionless parameters such as Strouhal number (St). For the fish swimming, the main characteristics of the wake are the reverse Kármán street and jet profile. This type of wake displays a convective instability [58]. The co-existence of the reverse Kármán street and jet profile is ensured only at the frequency of maximum-wake amplification at which the optimal efficiency is obtained. Specifically, for the small Strouhal number ($\mathrm{St} < 0.2$), the vortices behind a flapping foil are usually staggered in a Kármán street style, while for the high Strouhal number ($\mathrm{St} > 0.25$), the wake is a reverse Kármán street [58,60]. It is known that at low Strouhal numbers a net drag force is produced, while at high Strouhal numbers, a net thrust force is obtained [37]. In addition, the Strouhal number of the maximum-wake amplification is in the range of 0.25–0.35 [58]. Inspired by the above work, we can expect that the propulsive performance will be improved by controlling the vorticity generation. A clue of this type of applications is implied in two recent studies [16,68], in which it was found that a cylinder with the downstream half made flexible to form an appropriate traveling transverse wave or windward suction leeward blowing control benefits by eliminating the vortex shedding and reducing the average drag. In addition, the wake interaction of multiple bodies will make the coupling motion more complicate; and hence the bodies may benefit by reducing the average drag [2,15,25,53–55].

In this work, we will study the swimming performance and vorticity structures of a mother–calf pair of fish using a space-time finite element method. The drag, root-mean-square value of lift, hydrodynamic power, and vorticity structure are analyzed in detail by systematically varying the relative position of the calf, Reynolds number, phase difference and phase velocity.

The rest of this paper is organized as follows. In Section 2, the physical problem, mathematical formulation and numerical method are described. The numerical results and discussion are then presented in Section 3. Finally, concluding remarks are provided in Section 4.

2. Physical definition and mathematical method

2.1. Physical problem and mathematical formulation

In this work, the body shape of the fish is approximated by a two-dimensional foil (NACA0012). Without loss of generality, the mother/calf size ratio is taken as 2, as shown in Fig. 1(a). The midlines of both foils are making a lateral oscillation in the form of a wave traveling in the streamwise direction, described by

$$y_i(x,t) = a_i(x)\sin[2\pi(x-ct)+\varphi_i], \qquad (1)$$

where the subscript $i = \mathtt{M}$ and $\mathtt{C}$ denoting, respectively, mother fish and calf fish, a_i is the amplitude of the wave, c is the phase velocity, and φ_i is the phase. To reasonably model the lateral motion of the backbone undulation of fish swimming, the amplitude a_i is approximated by a quadratic polynomial,

$$a_i = C_0 + C_1 x + C_2 x^2,$$

where C_0, C_1 and C_2 are determined by $a_i(0 * L_i) = 0.02L_i$, $a_i(0.2 * L_i) = 0.01\ L_i$ and $a_i(1.0 * L_i) = 0.1\ L_i$ with L_i being the chord length of the model [12,62]. The positions of the calf relative to the mother is denoted by (x_c, y_c). Fig. 1(b) shows five typical positions of the calf relative to the mother. Although we do recognize the limitation of the assumptions, i.e. fixed size, identical locomotion and simple shape, we nevertheless feel that the results will be of help in understanding of the swimming performance and vorticity structures of a mother–calf pair of fish. Actually, similar models have been extensively used in this area [5–7,12,13,15,52].

The fluid dynamics is governed by the two-dimensional incompressible Navier–Stokes equations. To nondimensionalize the equations, the chord length of the mother fish L_M is used as the length scale, the free-stream velocity U as the velocity scale, and ρU^2 as the