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Interfacial deformation and jetting of a magnetic fluid

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ABSTRACT

An attractive technique for forming and collecting aggregates of magnetic material at a liquid–air interface by an applied magnetic field gradient was recently proposed, and its underlying principle was studied theoretically and experimentally (Tsai et al., 2013): when the magnetic field is weak, the deflection of the liquid–air interface has a steady shape, while for sufficiently strong fields, the interface destabilizes and forms a jet that extracts magnetic material. Motivated by this work, we develop a numerical model for the closely related problem of solving two-phase Navier–Stokes equations coupled with the static Maxwell equations. We computationally model the forces generated by a magnetic field gradient produced by a permanent magnet and so determine the interfacial deflection of a magnetic fluid (a pure ferrofluid system) and the transition into a jet. We analyze the shape of the liquid–air interface during the deformation stage and the critical magnet distance for which the static interface transitions into a jet. We draw conclusions on the ability of our numerical model to predict the large interfacial deformation and the consequent jetting, free of fitting parameters.

1. Introduction

Synthesis and assembly on the nanoscale is an important goal of contemporary science and technology. Magnetic nano/microparticles arise in a wide range of industrial and biomedical applications, and so are one target for controlled assembly. For example, functionalized magnetic microparticles can be used to separate cells [1] and magnetic microparticles have been used in microfluidics for cell sorting, blood cleansing, and magneto-capillary self-assembly (see e.g. [2] and references therein). When magnetic nanoparticles such as magnetite are suspended at high concentration in aqueous or non-aqueous carrier fluids, the entire system behaves as a continuum of magnetic fluid, known also as a ferrofluid. The rheology and interfacial shape of ferrofluids can be tuned with external magnetic fields, often in useful ways. An example is the application of ferrofluids in adaptive optics that has been considered in recent experiments [3,4]. The control of ferrofluid properties using magnetic fields also has applications in mechanical sealing and acoustics [5], targeted drug delivery [6–8] and treatment of retinal detachment [9].

Thin liquid films and droplets are ubiquitous in nature and also appear in many technological applications. The understanding of their dynamical behavior and their stability is therefore of great importance and has attracted considerable attention in the literature. Recent research into thin film and droplet flows has resulted in many experimental and theoretical developments, including manipulating film flows via external magnetic or electric fields to produce nanoscale patterns. In particular, experiments on thin ferrofluid films and droplets have revealed the formation of a wide range of morphologies [10–14]. Ferrofluids can be manipulated using magnetic forces and have been extensively investigated and widely used in a variety of engineering applications; see Rosensweig [15] and a more recent review by Nguyen [16]. Normal field instability of ferrofluid films (and the equivalent electric field problem) have been extensively studied in the past, see e.g. [17,18]. However, despite the increase in the number of applications, surprisingly little can be found in the literature on the direct numerical simulations of thin ferrofluid films in the presence of a nonuniform magnetic field (such as is produced by a spherical magnet) and therefore our understanding of the instabilities that may occur in these flows is limited.

An attractive technique for forming and collecting aggregates of magnetic material at a liquid–air interface by an applied magnetic field was recently proposed, and its underlying principle was studied theoretically and experimentally, by Tsai et al. [19]. In the experiments described in [19], a water-based ferrofluid (EMG805, Ferrotec), with a density of 1200 kg m^{-3} and viscosity of 3 mPa s, is suspended in a shallow reservoir containing deionized water, with a density of 1000 kg m^{-3} and viscosity of 1 mPa s, to form

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the magnetic mixture. This system is differentiated from a pure ferrofluid system because, in the presence of a magnet, it separates into a region rich with magnetic material, and one of negligible magnetic content. A spherical permanent magnet is slowly brought close to the magnetic mixture allowing the ferrofluid to aggregate and form a static hump at the liquid–air interface (see Fig. 1 in [19]). In these experiments, a distinct boundary that separates the magnetic and non-magnetic regions is observed. When the magnet is held sufficiently close to the liquid–air interface, the hump destabilizes and transforms to a jet. The theoretical approach developed in [19] describes a steady-state mathematical model for the behavior of the magnetic-particle-laden fluid and the particle-free fluid regions. The mathematical model results in [19] show excellent agreement with the experimental data.

Motivated by this work, here we develop a numerical model for a closely related problem: we computationally model the magnetically induced interfacial deflection of a magnetic fluid (ferrofluid) and the transition into a jet by a magnetic field gradient from a permanent magnet placed above the free surface. The system we study differs from that considered by Tsai et al. [19]: we consider a pure ferrofluid system, while Tsai et al. model a system with both magnetically dominated and non-magnetic regions. Fig. 1 shows a schematic illustrating the set-up we consider: the magnetic region occupied by pure ferrofluid, the liquid–air interface, and the spherical permanent magnet. The deformation of the ferrofluid–air interface arises as a result of the magnetic field gradient induced by the spherical permanent magnet held above the fluid; in line with the experiments we will see that, for sufficiently strong fields, the interface in our model destabilizes and forms a jet. We note that, although the focus of this work is to use a numerical study to uncover the transition to instability in a pure ferrofluid system, we believe that our study demonstrates some, perhaps not obvious, features of the development of the instability observed in the work of Tsai et al. [19]. The natural next step would be to consider the effect of the nonuniform distribution of the ferrofluid/magnetic particles, but this is beyond the scope of the present paper.

Here we solve the two-phase Navier–Stokes equations coupled with the static Maxwell equations in axisymmetric cylindrical polar coordinates. We analyze the shape of the liquid–air interface during the deformation stage and the critical magnet distance (from the undeformed free surface), for which the static interface transitions into a jet. We draw conclusions regarding the ability of our numerical model to predict the large interfacial deformation and the consequent jetting, free of fitting parameters. The numerical model provides a realistic and accurate framework for predicting the evolution of magnetic liquids based on the Navier–Stokes equations.

We describe the details of the numerical model in Section 2. In Section 3, we describe a numerical boundary condition that may be implemented to simulate non-uniform magnetic fields. In Section 4, we present the numerical results and the comparison with experimental observations. In Section 5, we give an overview and future outlook for improving our modeling.

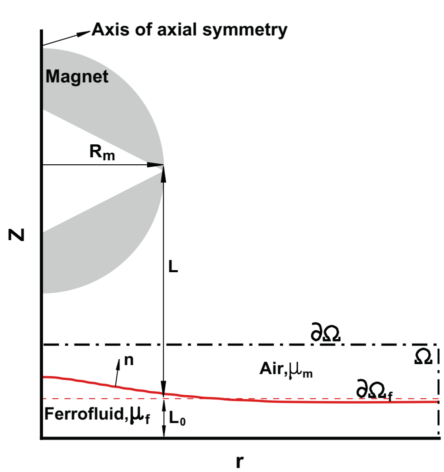

Fig. 1. A schematic illustrating the computational set-up and the coordinate system used. A spherical permanent magnet of radius R_m is centered at distance L from the initially undeformed film (red dashed line), which has a depth L_0. The magnetic force deforms the interface, $\partial\Omega_f$, into a hump (red solid line). A unit normal outwards from the interface is denoted by **n**. A typical computational domain, Ω, and its boundary, $\partial\Omega$, is shown by the dash-dotted line. We use axisymmetric cylindrical polar coordinates. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. Mathematical model

Here we briefly describe the theoretical models that serve as a basis for the proposed numerical studies. The coupled motion of a ferrofluid surrounded by a non-magnetic fluid is governed by the (static) Maxwell equations, the Navier–Stokes equations, and a constitutive relationship for the magnetic induction **B**, magnetic field **H**, and the magnetization **M**. The magnetostatic Maxwell equations for a non-conducting ferrofluid are, in SI units,

$$\nabla\cdot\mathbf{B}=0,\quad \nabla\times\mathbf{H}=0,\quad \mathbf{B}(\mathbf{x},t)=\begin{cases}\mu_f\mathbf{H} & \text{in ferrofluid}\\ \mu_m\mathbf{H} & \text{in matrix,}\end{cases}$$

where μ_f denotes the magnetic permeability of the ferrofluid and μ_m is the permeability of the matrix fluid. For our application, the matrix fluid is air, which has a permeability very close to that for a vacuum, μ_o. Therefore, we shall consider $\mu_m=\mu_o$ throughout. A magnetic scalar potential ψ is defined by $\mathbf{H}=\nabla\psi$, and satisfies

$$\nabla\cdot(\mu\nabla\psi)=0, \tag{1}$$

where $\mu=\mu_o$ and μ_f in the matrix and ferrofluid, respectively. We will assume that the magnetization is a linear function of the magnetic field given by $\mathbf{M}=\chi\mathbf{H}$, where $\chi=(\mu_f/\mu_o-1)$ is the magnetic susceptibility [20]. The magnetic induction **B** is therefore $\mathbf{B}=\mu_o(\mathbf{H}+\mathbf{M})=\mu_o(1+\chi)\mathbf{H}$.

The fluid equation of motion is described by the conservation of mass and momentum (Navier–Stokes) equations

$$\nabla\cdot\mathbf{u}=0, \tag{2}$$

$$\rho\left(\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}\right)=-\nabla p+\nabla\cdot(2\eta\mathbf{D})+\nabla\cdot\tau_m+\mathbf{F}_s+\rho\mathbf{g}, \tag{3}$$

where $\mathbf{F}_s$ denotes the surface tension force per unit volume (presented as a body force [21]), p is pressure, **u** is velocity, $\mathbf{D}=\frac{1}{2}(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)$ is the rate of deformation tensor (where T denotes the transpose), η is viscosity, ρ is density, τ_m is the magnetic stress tensor, and **g** is the gravitational acceleration. The total stress is $\tau=-p\mathbf{I}+2\eta\mathbf{D}+\tau_m$, where **I** denotes the identity operator. The magnetic stress tensor of an incompressible, isothermal, magnetizable medium is [22]

$$\tau_m=-\frac{\mu_o}{2}H^2\mathbf{I}+\mu\mathbf{H}\mathbf{H}^T,$$

where $H=|\mathbf{H}|$. These equations must be solved subject to suitable boundary and initial conditions, discussed in Section 3 below.

3. Numerical methodologies

We will use an Eulerian framework, where the material moves through a stationary mesh, and therefore a special procedure will