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Calibration of a viscoplastic cohesive zone for crazing in PMMA

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Abstract

In a numerical analysis of mode I fracture in amorphous polymers, Estevez et al. [Estevez R, Tijssens MGA, Van der Giessen E. Modelling of the competition between shear yielding and crazing in glassy polymers. J Mech Phys Solids 2000;48:2585–617] have shown that the material toughness is governed by the competition between the time scales related to shear yielding and crazing. The present study aims at calibrating the parameters involved in this description, for a commercial PMMA. An elastic–viscoplastic constitutive law featuring softening upon yielding and hardening at continued deformation is used for the bulk while crazing is described with a viscoplastic cohesive zone. The three steps of crazing with initiation for a critical stress state, thickening of the craze surfaces and breakdown of the craze fibrils for a critical opening are characterized separately. In particular, it is demonstrated that the use of a viscoplastic cohesive zone is necessary to capture the variation of the toughness with loading rate. For PMMA, the related energy release rate is shown to depend primarily on the craze critical opening and the craze thickening kinetics while craze initiation is of minor importance for the quasi-static loading conditions under consideration here.

Keywords: Crazing; Cohesive zone; PMMA; Elastic–viscoplastic; Fracture

1. Introduction

Failure of amorphous polymers in the glassy state involves two mechanisms of damage and failure: shear yielding and crazing [1]. When crazing is suppressed, as in compression, shear yielding takes place in the form of a localized plastic deformation through shear bands related to the intrinsic softening upon yielding followed by a progressive strain hardening as the deformation continues. Crazing involves also some localized plasticity [2,3], albeit at a smaller scale, and is the mechanism responsible for failure. After initiation at a critical stress state, the craze thickens by the growth of fibrils of which breakdown at a critical thickness corresponds to the nucleation of a crack. In a numerical study [1] featuring a viscoplastic model for shear yielding and a

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viscoplastic cohesive zone for crazing, it was demonstrated that the competition between the kinetics of these two mechanisms together with the condition for the craze fibrils breakdown govern the level of the toughness, as for instance the ductile to brittle transition observed at low loading rates. A ductile response is related to the development of some plasticity in the bulk prior to crack propagation while a brittle response corresponds to the development of crazing only, the bulk remaining elastic. The present study is connected to a modelling of crazing [1,4] within a cohesive surface methodology which incorporates the three characteristic stages of crazing. Estevez et al. have shown [1] that the description adopted here is able to capture *qualitatively* the main features of amorphous polymers fracture as the rate dependency of the toughness. The aim of the present study is to provide some *quantitative* estimates of the fracture characteristics and to identify the dominant mechanism or effect responsible for the material's toughness at a given loading condition. We present the complete experimental protocol necessary to calibrate the parameters of the description of crazing and shear yielding. The calibration is based on the analysis of fracture and uniaxial compression tests performed at various loading rates, for quasi-static loading conditions. It is shown that a viscoplastic cohesive zone model for crazing is necessary to predict the rate dependent toughness observed experimentally. Experimental measurements of the toughness at various loading rates in specimens with two different blunted crack tips appear to be in good agreement with the corresponding predictions. In particular, the model is able to capture the size effect introduced when varying the notch radius thanks to the intrinsic length scale introduced by the cohesive zone description.

In order to illustrate the methodology, a commercial PMMA (Perspex) is used, which is generally thought brittle and (quasi-) linear elastic under tension. While the bulk response of PMMA can be considered linear elastic at a first approximation but with a secant Young's modulus representing the viscoelastic effects, we show that its fracture characteristics are noticeably rate dependent.

Tensors are denoted by bold-face symbols, $\otimes$ is the tensor product and $\bullet$ the scalar product. For example, with respect to a Cartesian basis $\boldsymbol{e}_i$, $\boldsymbol{AB} = A_{ik}B_{kj}\boldsymbol{e}_i \otimes \boldsymbol{e}_j$, $\boldsymbol{A} \bullet \boldsymbol{B} = A_{ij}B_{ij}$ and $\mathscr{L}\boldsymbol{B} = L_{ijkl}B_{kl}\boldsymbol{e}_i \otimes \boldsymbol{e}_j$, with an implicit summation over Latin indices. The summation is *not* used for repeated Greek indices. A $(\,)'$ identifies the deviatoric part of a second-order tensor, $\boldsymbol{I}$ is the identity second-order tensor and tr denotes the trace.

2. Bulk constitutive law

2.1. Modelling background

In the absence of crazing, glassy polymers can undergo a deformation up to 100% with an intriguing constitutive law with softening upon yielding followed by hardening. In an analysis of the crack tip plasticity under a mode I loading, Van der Giessen and Lai [5] have shown that the observed softening is intrinsic to the material response and is not due to a structure or geometrical effect. Their prediction of the shape of the plastic zone and trajectories of the shear bands is in good agreement with reported observations of Ishikawa et al. [6]. The constitutive law used to model the large strain plastic behaviour is based on original ideas due to Boyce et al. [7] but with some modifications introduced later by Wu and Van der Giessen [8] for the hardening part. We present the governing equations to point out the parameters to be identified, the reader is referred to [9] for details on the computational aspects.

The mechanics of fully three-dimensional large strain deformation involves the deformation gradient tensor $\boldsymbol{F}$, which maps a material point of the reference configuration into the current configuration. The deformation gradient is multiplicatively decomposed as $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}}\boldsymbol{F}^{\mathrm{p}}$, with $\boldsymbol{F}^{\mathrm{p}}$ a deformation from the initial to an intermediate, ''relaxed'' or ''natural'' configuration, followed by an elastic transformation $\boldsymbol{F}^{\mathrm{e}}$ up to the final deformation $\boldsymbol{F}$. The velocity gradient in the current configuration is $\boldsymbol{L} = \dot{\boldsymbol{F}}\boldsymbol{F}^{-1} = \dot{\boldsymbol{F}}^{\mathrm{e}}\boldsymbol{F}^{-\mathrm{e}} + \boldsymbol{F}^{\mathrm{e}}\dot{\boldsymbol{F}}^{\mathrm{p}}\boldsymbol{F}^{-\mathrm{p}}\boldsymbol{F}^{-\mathrm{e}}$ of which symmetric and anti-symmetric part correspond to the rate of the strain and spin tensor respectively. When the elastic part $\boldsymbol{F}^{\mathrm{e}}$ of the deformation gradient is small compared to the plastic one $\boldsymbol{F}^{\mathrm{p}}$ (i.e. $\boldsymbol{F}^{\mathrm{e}} \approx \boldsymbol{I}$), the velocity gradient results in $\boldsymbol{L} \approx \boldsymbol{L}^{\mathrm{e}} + \boldsymbol{L}^{\mathrm{p}}$ so that the total strain rate $\boldsymbol{D}$ becomes the sum of the elastic and plastic parts as $\boldsymbol{D} = \boldsymbol{D}^{\mathrm{e}} + \boldsymbol{D}^{\mathrm{p}}$. Prior to the yield stress, most amorphous polymers show a non-linear stress–strain response due to small viscoelastic effects. These are not considered explicitly but their effect on the mechanical response is accounted for by using a secant Young's modulus instead of that derived from ultra-sonic measurements of the elastic wave velocities. Its value is estimated from uniaxial compression tests and an average value for dif-