Computers & Fluids 97 (2014) 126–142

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A simple immersed-boundary method for solid–fluid interaction in constant- and stratified-density flows

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ARTICLE INFO

Article history:
Received 17 March 2013
Received in revised form 18 February 2014
Accepted 27 March 2014
Available online 18 April 2014

Keywords:
Immersed-boundary method
Stratified density flows
Navier–Stokes equations
Solid–fluid interaction

ABSTRACT

The present work reports on the simulation of two- and three-dimensional constant- and stratified-density flows involving fixed or moving objects using an immersed-boundary method. The numerical approach is based on a simple immersed-boundary method in which no explicit Lagrangian marking of the immersed boundary is used. The solid object is defined by a continuous solid volume fraction which is updated thanks to the resolution of the Newton's equations of motion for the immersed object. As shown on several test cases, this algorithm allows the flow field near the solid boundary to be correctly captured even though the numerical thickness of the transition region separating the fluid from the object is within three computational cells approximately. The full set of governing equations is then used to investigate some fundamental aspects of solid–fluid interaction, including fixed and moving objects in constant and stratified-density flows. In particular, the method is shown to accurately reproduce the steady-streaming patterns observed in the near-region of an oscillating sphere, as well as the so-called Saint Adrew's cross in the far-field when the sphere oscillates in a rotating stratified fluid. The sedimentation of a particle in a stratified ambient is investigated for particle Reynolds numbers up to $\mathcal{O}(10^3)$ and the effect of stratification and density ratio is addressed. While the present paper only consider fluid–solid interaction for a single object, the present approach can be straightforwardly extended to the case of multiple objects of arbitrary shape moving in a stratified-density flow.

1. Introduction

Solid–fluid interactions are encountered in a large number of industrial and natural applications, including chemical engineering, aeronautics, transportations, biomecanics, geophysics and oceanography, to name but a few. Modeling solid–fluid interaction is often difficult because of the complexity of the solid shape and motion in the fluid flow. Reproducing the dynamics of multiple interacting objects of arbitrary geometry with possible deformation is made even more challenging if the flow is non-uniform in composition (multiphase flows), density (compressible or stratified flows) or temperature (heat transfer, phase change).

Methods for modeling solid–fluid interaction may be divided within two main groups, depending on the way the solid–fluid interfaces are described. One group, usually referred to as "body-fitted grid methods" makes use of a structured curvilinear or unstructured grid to conform the grid to the boundary of the fluid domain (see e.g. [59,41] for grid generation techniques). In situations involving complex moving boundaries, one needs to establish a new body-conformal grid at each time-step which leads to a substantial computational cost and subsequent slowdown of the solution procedure. In addition, issues associated with regridding arise such as grid-quality and grid-interpolation errors.

The second group of methods is referred to as "fixed-grid methods". These techniques make use of a fixed grid, which eliminates the need of regridding, while the presence of the solid objects is taken into account via adequately formulated source terms added to fluid flow equations. Fixed-grid methods have emerged in recent years as a viable alternative to body-conformal grid methods. In this group, one can mention distributed Lagrange multiplier with a fictitious-domain (DLM) based methods [23,51,50,65,2], immersed-boundary method (IBM) [54,20,37,63], lattice Boltzmann method (LBM) [38], penalty method [36,56] and ghost-fluid method [21] have been developed and shown to be effective in computing fluid-particle systems and fluid-structure interaction problems.

The immersed-boundary method was first introduced by Peskin [53] for computing blood flow in the cardiovascular system. In the

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http://dx.doi.org/10.1016/j.compfluid.2014.03.030
0045-7930/

original method the flow field is described on a fixed Eulerian grid and the immersed boundary is represented with a set of Lagrangian points on which the no-slip boundary condition is enforced by adding appropriate boundary forces. The boundary forces which are singular Dirac functions along the surface in the continuous equations are described by discrete regularization functions that smear the forcing effect over the neighboring Eulerian cells (see e.g. Fig. 2 of [44]). The immersed-boundary method has been improved since the pioneering work of Peskin and many variants can be found in the literature ([43,1], see also the reviews of [34,44]).

While immersed-boundary method has been used in a wide range of applications (compressible flows, particulate flows, micro-scale flows, multi-phase flows, conjugate heat transfer, see e.g. Kang et al. [35] and reference therein), application to stratified flows has been rare. To our knowledge, the only recently reported work using a fixed-grid approach computing the motion of rigid objects in a stratified fluid is that of Doostmohammadi and Ardekani [19] who used a DLM approach to investigate the interaction of a pair of particles sedimenting in a stratified fluid, using the Boussinesq approximation. Here we present an immersed-boundary method aimed at describing the motion of multiple objects of arbitrary shape in a constant- or stratified-density flow. The specificity of the present method is that (i) the treatment of the solid–fluid interaction is simple and easy to implement in the sense that there is no Lagrangian marking of the immersed boundary nor interpolation needed and (ii) the fluid density can be inhomogeneous, with no restriction on the density gradient, i.e. the method is applicable to non-Boussinesq flows. Details of the numerical scheme are outlined in Section 2 and the method is applied to investigate solid–fluid interaction in constant and stratified-density flows in Section 3, in which both forced motion and freely moving rigid objects are simulated.

2. Governing equations and numerical method

2.1. Governing equations and assumptions for the fluid phase

Assuming a variable-density non-diffusive Newtonian fluid, the evolution of the flow is then described using the Navier–Stokes equations, namely

$$\frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot (\mathbf{V} \otimes \mathbf{V}) = \mathbf{g} - \frac{1}{\rho}\nabla P + \frac{1}{\rho}\nabla \cdot [\mu(\nabla \mathbf{V} + \nabla \mathbf{V}^T)] + \mathbf{f}, \tag{1}$$

$$\nabla \cdot \mathbf{V} = 0. \tag{2}$$

In (1) and (2), $\mathbf{V}, P, \rho$ and μ denote the local velocity, pressure, density and viscosity of the fluid, respectively, $\mathbf{g}$ denotes gravity and $\mathbf{f}$ is a volume force term used to take into account solid–fluid interaction. The local density of the non-diffusive fluid obeys

$$\frac{\partial \rho}{\partial t} + (\mathbf{V} \cdot \nabla)\rho = 0. \tag{3}$$

The detailed development of 1, 2 in the more general case of diffusive fluids can be found in Cook and Dimotakis [15]. Here, we simply set the diffusivity to zero. 1, 2 are written in a general system of orthogonal curvilinear coordinates. However, in the present work, only Cartesian or polar systems of coordinates were used. The reader is referred to Magnaudet et al. [42] for more details about the resolution of (1) and (2) in the more general system of orthogonal curvilinear coordinates.

Eqs. (1)–(3) are enforced throughout the entire domain, including the actual fluid domain and the space occupied by the immersed boundary. In the following, the term $\mathbf{f}$ will be formulated such as to represent the action of the immersed solid boundaries upon the fluid.

Here, we consider a fluid of variable density for which non-Boussinesq effects may play a role. In the general case of diffusive non-Boussinesq fluids there are some fundamental issues with the proper governing equations to be used. As discussed in Joseph and Renardy [29] and Chen and Meiburg [13] among others, divergence effects and Korteweg stresses can potentially be important in regions of large concentration gradients and need to be taken into account in physical models. These effects do not need to be included if one assumes the fluid to be non-diffusive, as in the present work. Conversely, using a non-diffusive fluid may results in sharp local density gradients which may cause computational difficulties, especially in the case of solid objects moving in a stratified fluid [60]. Here, such issues are circumvented by the use of a numerical scheme specifically designed to handle sharp gradients for the equation of transport of density, as described in Section 2.3.

2.2. Equations of motion for the solid phase

Let us consider a non-deformable solid object of density ρ_p and volume ϑ_p, the centroid of which being located at $\mathbf{x}_p$, moving at linear and angular velocity $\mathbf{u}_p$ and ω_p, respectively. Here the index "p" refers to particle label. The local velocity in the object is then defined by $\mathbf{U} = \mathbf{u}_p + \omega_p \times \mathbf{r}, \mathbf{r}$ being the local position relative to the solid centroid. As will be detailed in the next section, the volume force $\mathbf{f}$ is chosen to ensure $\mathbf{V} = \mathbf{U}$ in ϑ_p (rigid-body motion throughout the volume of the solid object). Thus, integrating momentum and kinematic momentum laws for the fluid on ϑ_p gives [63]

$$\frac{d}{dt}\int_{\vartheta_p} \rho \mathbf{V} d\vartheta = \bar{\rho}\vartheta_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \tau \cdot \mathbf{n} dS + \int_{\vartheta_p} \rho \mathbf{f} d\vartheta + \bar{\rho}\vartheta_p \mathbf{g}, \tag{4}$$

$$\frac{d}{dt}\int_{\vartheta_p} \rho \mathbf{r} \times \mathbf{V} d\vartheta = \bar{\rho}\mathbf{I}_p \frac{d\omega_p}{dt} = \int_{S_p} \mathbf{r} \times (\tau \cdot \mathbf{n}) dS + \int_{\vartheta_p} \rho \mathbf{r} \times \mathbf{f} d\vartheta, \tag{5}$$

with $\tau = -P\mathbf{I} + \mu(\nabla \mathbf{V} + \nabla \mathbf{V}^T)$ being the hydrodynamic stress tensor, $\mathbf{I}_p$ the inertia matrix, $\mathbf{n}$ the outward-pointing normal vector on the solid–fluid boundary S_p and $\bar{\rho}$ the averaged fluid density in the volume occupied by the particle, *viz*

$$\bar{\rho} = \frac{1}{\vartheta_p}\int_{\vartheta_p} \rho d\vartheta. \tag{6}$$

Note that in the case of a constant-density fluid $\bar{\rho} = \rho$. The motion of the solid object can be either externally imposed or driven by its weight and the fluid forces on its boundary. In the latter case, it is described by Newton's equations for linear and angular momentum of a rigid body, namely

$$\rho_p \vartheta_p \frac{d\mathbf{u}_p}{dt} = \int_{S_p} \tau \cdot \mathbf{n} dS + \rho_p \vartheta_p \mathbf{g}, \tag{7}$$

$$\mathbf{I}_p \frac{d\omega_p}{dt} = \int_{S_p} \mathbf{r} \times (\tau \cdot \mathbf{n}) dS. \tag{8}$$

In order to ensure that the fictitious body force $\mathbf{f}$ is such that (7) and (8) are equivalent to (4) and (5), respectively, we obtain the following equations of motion *viz*

$$\frac{d\mathbf{u}_p}{dt} = \mathbf{g} - \frac{1}{(\rho_p - \bar{\rho})\vartheta_p}\int_{\vartheta_p} \rho \mathbf{f} d\vartheta, \tag{9}$$

$$\mathbf{I}_p \frac{d\omega_p}{dt} = -\frac{\rho_p}{(\rho_p - \bar{\rho})}\int_{\vartheta_p} \rho \mathbf{r} \times \mathbf{f} d\vartheta. \tag{10}$$

2.3. Spatial discretization and time-integration of the full system of equations

Our computational procedure employs a finite-volume approach on a staggered grid [31]. The transport equation of the