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Visualization of three-dimensional incompressible flows by quasi-two-dimensional divergence-free projections

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ABSTRACT

Context: A visualization of three-dimensional incompressible flows by divergence-free quasi-two-dimensional projections of velocity field on three coordinate planes is proposed.

Objective: To visualize 3D incompressible flow by 3 two-dimensional plots.

Method: It is argued that such divergence-free projections satisfying all the velocity boundary conditions are unique for a given velocity field. It is shown that the projected fields and their vector potentials can be calculated using divergence-free Galerkin bases.

Results: Using natural convection flow in a laterally heated cube as an example, it is shown that the projections proposed allow for a better understanding of similarities and differences of three-dimensional flows and their two-dimensional likenesses. An arbitrary choice of projection planes is further illustrated by a lid-driven flow in a cube, where the lid moves parallel either to a sidewall or a diagonal plane.

Conclusion: A new method for visualization of 3D incompressible flows is developed and described.

1. Introduction

With the growth of available computer power, development of numerical methods and experimental techniques dealing with fully developed three-dimensional flows the importance of flow visualization becomes obvious. While two-dimensional flows can be easily described by streamline or vector plots, there is no commonly accepted methodology for representation of three-dimensional flows on a 2D plot. Streamlines can be defined also for a general 3D flow, however cannot be represented by a single stream function. Other textbook techniques, such as streak lines, trajectories and arrow fields, are widely used but become unhelpful with increase of flow complexity. Same can be said about plotting of isosurfaces and isolines of velocity or vorticity components, which produce beautiful pictures, however, do not allow one to find out velocity direction at a certain point. Basic and more advanced recent state-of-the-art visualization techniques are discussed in review papers [1–3] where reader is referred for the details. Here we develop another visualization technique, applicable only to incompressible flows, and related to the surface-based techniques discussed in [2]. Our technique considers projections of 3D velocity field onto coordinate planes and allows one to compute a set of surfaces to which the projected flow is tangent. Thus, the flow is visualized in all three sets of coordinate planes (surfaces). The choice of visualization coordinate system is arbitrary, so that the axes can be directed along "most interesting" directions, e.g. directions parallel and orthogonal to dominating velocity or vorticity.

The goal of this study is to visualize a three-dimensional incompressible flow computed numerically at some grid nodes. The visualization described below is independent on the method used for flow calculation. It is based on divergence-free projections of a computed 3D velocity field on two-dimensional coordinate planes. Initially, this study was motivated by a need to visualize three-dimensional benchmark flows, which are direct extensions of well-known two-dimensional benchmarks, e.g., lid-driven cavity and convection in laterally heated rectangular cavities. Thus, we seek for a visualization that is capable to show clearly both similarities and differences of flows considered in 2D and 3D formulations. It seems, however, that the technique proposed can have significantly wider area of applications and can be applied for visualization of different divergence free vector functions, e.g., vorticity and magnetic field. The last example below illustrates that the technique allows one to visualize along or perpendicular to an arbitrarily chosen direction that can differ from coordinate axes.

Consider a given velocity field, which can be a result of computation or experimental measurement. Note, that modern means of flow measurement, like PIV and PTV, allow one to measure three velocity components on quite representative grids, which leads to the same problem of visualization of results. Here we observe that a three-dimensional divergence-free velocity field can be represented as a superposition of two vector fields that describe the

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motion in two sets of coordinate planes, say (x–z) and (y–z), without a need to consider the (x–y) planes. These fields allow for definition of vector potential of velocity, whose two independent components have properties of two-dimensional stream function. The two parts of velocity field are tangent to isosurfaces of the vector potential components, which allows one to visualize the flow in two sets of orthogonal coordinate planes. This approach, however, does not allow one to preserve the velocity boundary conditions in each of the fields separately, so that some of the boundary conditions are satisfied only after both fields are superimposed. The latter is not good for visualization purposes. We argue further, that it is possible to define divergence-free projections of the flow on the three sets of coordinate planes, so that (i) the projections are unique, (ii) each projection is described by a single component of its vector potential, and (iii) the projection vectors are tangent to isosurfaces of the corresponding non-zero vector potential component. This allows us to visualize the flow in three orthogonal sets of coordinate planes. In particular, it helps to understand how the three-dimensional model flows differ from their two-dimensional likenesses. To calculate the projections we propose to use divergence-free Galerkin bases, on which the initial flow can be orthogonally projected. Clearly, these projections can be calculated by other numerical approaches.

For a representative example, we choose convection in a laterally heated square cavity with perfectly thermally insulated horizontal boundaries, and the corresponding three-dimensional extension, i.e., convection in a laterally heated cube with perfectly insulated horizontal and spanwise boundaries. The most representative solutions for steady states in these model flows can be found in [4] for the 2D benchmark, and in [5,6] for the 3D one. In these benchmarks the pressure p, velocity $\boldsymbol{v} = (u, v, w)$ and temperature T are obtained as a solution of Boussinesq equations

$$\frac{\partial T}{\partial t} + (\boldsymbol{v} \cdot \nabla) T = \Delta T \qquad (1)$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} = -\nabla p + Pr \Delta \boldsymbol{v} + Ra Pr T \boldsymbol{e}_z \qquad (2)$$

$$div[\boldsymbol{v}] = 0 \qquad (3)$$

defined in a square $0 \leqslant x, z \leqslant 1$ or in a cube $0 \leqslant x,y,z \leqslant 1$, with the no-slip boundary conditions on all the boundaries. The boundaries $x = 0, 1$ are isothermal and all the other boundaries are thermally insulated, which in the dimensionless formulation reads

$$T(x=0) = 1, \quad T(x=1) = 0, \quad \left(\frac{\partial T}{\partial y}\right)_{y=0,1} = 0, \quad \left(\frac{\partial T}{\partial z}\right)_{z=0,1} = 0. \qquad (4)$$

Ra and Pr are the Rayleigh and Prandtl numbers. The reader is referred to the above cited papers for more details. Here we focus only on visualization of solutions of 3D problem and comparison with the corresponding 2D flows. All the flows reported below are calculated on 100^2 and 100^3 stretched finite volume grids, which is accurate enough for present visualization purposes (for convergence studies see also [7]).

Apparently, the 2D flow $\boldsymbol{v} = (u, 0, w)$ is best visualized by the streamlines, which are the isolines of the stream function ψ defined as $u = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial x}$. In each point the velocity vector is tangent to a streamline passing through the same point, so that plot of streamlines and schematic indication of the flow direction is sufficient to visualize a two-dimensional flow. This is illustrated in Fig. 1, where streamlines of flows calculated for $Pr = 0.71$, and Ra varied from 10^3 to 10^8 are shown. Note how the streamline patterns get more complex with the increase of Rayleigh number. Our further purpose is to visualize three-dimensional flows at

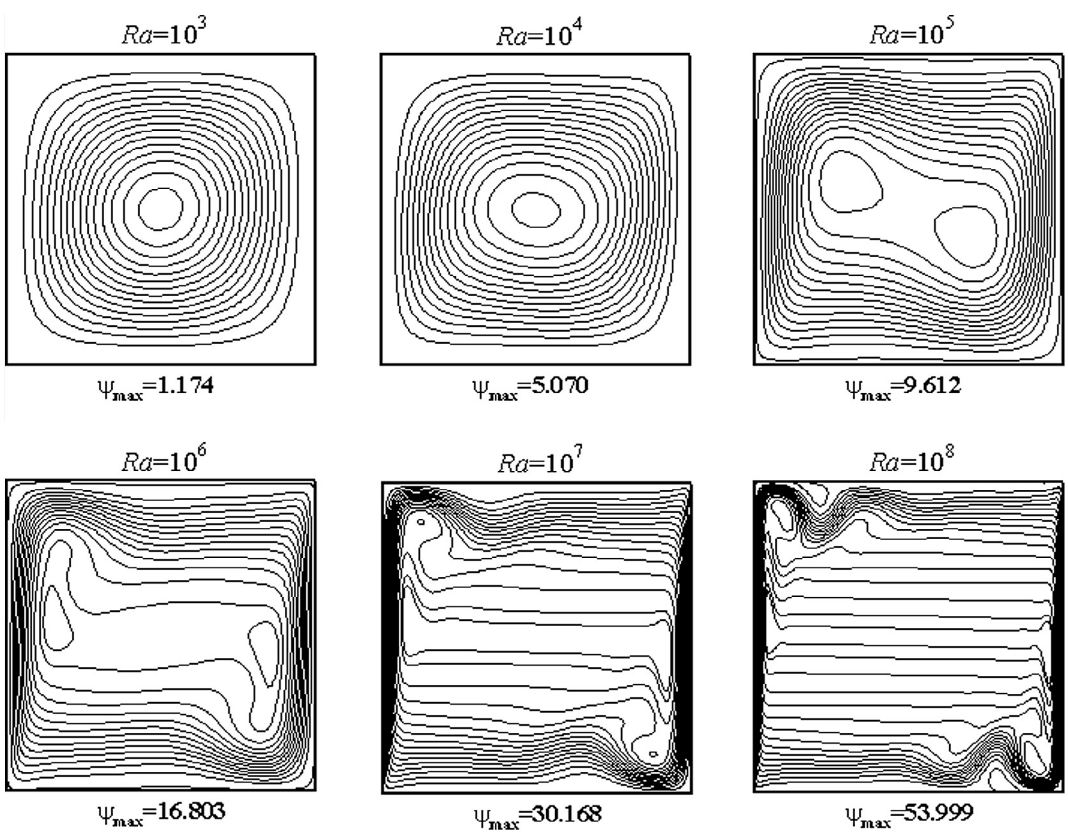

Fig. 1. Streamlines of two-dimensional buoyancy convection flow in a laterally heated square cavity at $Pr = 0.71$ and different Rayleigh numbers. The direction of main circulation is clockwise.