Engineering Failure Analysis 59 (2016) 211–222

Contents lists available at ScienceDirect

Engineering Failure Analysis

journal homepage: www.elsevier.com/locate/efa

Stochastic analysis on flexural behavior of reinforced concrete beams based on piecewise response surface scheme

Tengfei Xu [a,b,c], Tianyu Xiang [a], Renda Zhao [a], Guotao Yang [b], Cheng Yang [a,c,*]

[a] Department of Bridge Engineering, Southwest Jiaotong Unviersity, Chengdu 610031, PR China
[b] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, UNSW, Sydney, NSW 2052, Australia
[c] Key Laboratory of High-speed Railway Engineering, Ministry of Education, Southwest Jiaotong University, Chengdu 610031, PR China

ARTICLE INFO

Article history:
Received 23 March 2015
Received in revised form 22 September 2015
Accepted 5 October 2015
Available online 14 October 2015

Keywords:
Reinforced concrete
Deflection
Random analysis
Response surface method
Creep and shrinkage

ABSTRACT

To save the computational efforts of Monte Carlo simulation together with nonlinear finite element method, the analysis framework combining the response surface method and Monte Carlo simulation is usually adopted to investigate the stochastic nonlinear behavior of structures. It is found that the traditional response surface method could not describe stochastic behavior of cracked concrete beams. In order to overcome the discontinuity caused by cracking nonlinearity, a scheme by introducing a piecewise response surface is proposed in this paper. This scheme is evaluated by the stochastic analysis of several concrete beams. The comparison between the proposed method and some traditional methods, including Monte Carlo simulation and Monte Carlo simulation with traditional response surface, shows that the proposed method could well depict the stochastic behavior of concrete beams. Finally, the probability density evolution of concrete beams from short-term deflection to a long-term deflection is analyzed.

1. Introduction

To ensure the serviceability of reinforced concrete structures, deflection control is very important in engineering practice, as excessive deformation is one of the most common causes of damage and could result in large annual cost to the construction industry [1]. Actually, both overestimating and underestimating deformation of the engineering structures could cause safety problem in engineering. For example, in order to maintain a satisfied elevation of a bridge, pre-camber should be set based on the deflection prediction under long-term dead loads and live loads. Either overestimating or underestimating the pre-camber of a bridge will lead to track irregularity in high-speed railway, which may result in discomfort to passengers and insecurity to trains. Therefore, predicting the deformation of concrete structure as exactly as possible is essential to the industry community.

Due to the natural uncertainties of mechanical characters of concrete [2], such as the modulus of elasticity, the compressive and tensile strength, and the creep and shrinkage parameters, a deterministic analysis approach for calculating the deflections of a concrete structure is likely difficult to predict the experimental-measured deflections. In 1979, Ramsay et al. [3] studied the statistical variation of the immediate deflection of reinforced concrete (RC) beams, and the obtained coefficient of variation was between 25% and 30%. Creep and shrinkage is another uncertainty material property of concrete structures [4,5]. In order to take into account the randomness, several numerical methods has been adopted by the researcher to investigate the stochastic response of concrete structures, such as Monte Carlo Simulation (MCS) method [2,3,6,7] and Latin Hypercube Simulation (LHS) method [5,8].

---

* Corresponding author at: Department of Bridge Engineering, Southwest Jiaotong Unviersity, Chengdu 610031, PR China.
E-mail address: yangcheng@swjtu.edu.cn (C. Yang).

http://dx.doi.org/10.1016/j.engfailanal.2015.10.004
1350-6307/© 2015 Elsevier Ltd. All rights reserved.

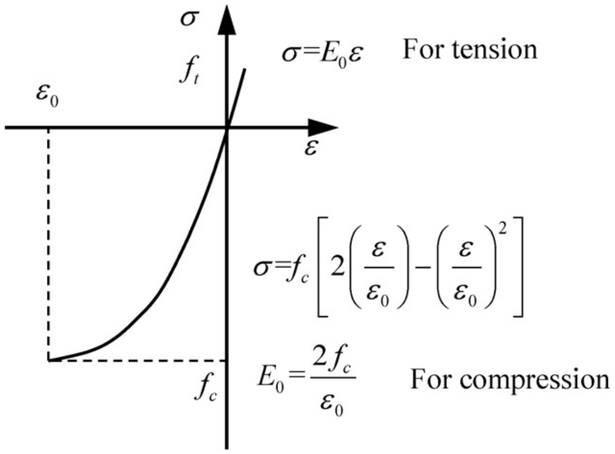

Fig. 1. Stress–strain relationship for concrete.

Monte Carlo Simulation method, which adopts a random sampling technique based on a computer-based analytical or numerical model, could provide the most reliable result in simulating the stochastic behavior of a structural system [9]. However, the MCS method takes huge computational cost, especially when the nonlinear finite element analysis is required in every sampling. Instead, the response surface method (RSM), which approximates the implicit response function by a simple and explicit polynomial, provides an effective and versatile way to applying numerical methods [9,10] The computing cost could be saved significantly in MCS if the structure response could be depicted by RSM.

It is worth noting that the major cause of the variability of the deflection is the variability of the beam stiffness, which is affected by cracking of the concrete and the variability in the concrete strength, particularly the tensile strength [3]. Due to the large discrepancy between cracked and uncracked stiffness [2], the real deflection response surface of a RC beam with respect to the tensile strength of the concrete may be unsmooth and convex, which could lead to the failure in estimating the random deflections if regular response surface method is adopted. To this end, a piecewise response surface method is proposed in this paper to solve the aforementioned problem, which is not reported yet in literature. By using the proposed method, the evolution of immediate deflection to long-term deflection of the test beams is also investigated.

2. Stochastic analysis model

2.1. Finite element formulations

The nonlinear finite element analysis should be carried out to reflect the actual loading-deflection behavior of concrete structures. A general finite element analysis program with degenerated beam element is adopted for immediate and long-term analysis [11]. The theory behind the degeneration technique has been widely used in the formulation of shell and plate element [12,13]. Based on the theory of Timoshenko beam, the three-dimensional displacement field of degenerated beam is described by the nodal variables. Meanwhile, the strain–displacement equations of three-dimensional structure are retained. To integrate the stiffness matrix and internal force, the piecewise integration method is usually used in the degenerated beam element [12,13]. In this method, the element is subdivided into several pieces, and one integration point is attributed for every piece. The stress of integration point can be computed following one-dimensional constitutive model. With this approach, the material nonlinearity of steel and concrete can be simulated conveniently. In the present study, the one-dimensional perfect elasto-plastic constitutive model is adopted for steel. Meanwhile, a nonlinear strain–stress relationship for concrete shown in Fig. 1 is introduced in the present study.

Table 1
Statistical properties of random variables.

Variables	Distribution type	Mean	Coefficient of variation	References
α_1	Normal	1.0	0.26	[5]
α_2	Normal	1.0	0.25	[5]
α_3	Normal	1.0	0.15	[21]
α_4	Normal	1.0	0.20	[4]
α_5	Normal	1.0	0.20	[22]
α_6	Normal	0.3	0.15	[20]