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Technical note

Validity of the spring-backed membrane model for bubble–wall interactions with compliant walls

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ABSTRACT

A numerical study was conducted to investigate the validity of using the spring-backed membrane model for compliant walls (CWs), and to explain the failure of the reference study (Duncan and Zhang, 1991) to reproduce the bubble behavior near moderately elastic CWs. The same mathematical models used in the reference study were employed in our work. The bubble was created from its minimum-volume state, and the initial liquid pressure on the CWs was computed, rather than artificially prescribed. Our predicted bubble behavior in the vicinity of moderately elastic CWs agrees qualitatively well with experimental observations. The spring-backed membrane model was demonstrated to be capable of enabling the correct behavior of CWs and the correct bubble dynamics in their vicinity. The improper choice/specification of the initial conditions in the reference study was determined to be responsible for the failure to reproduce the bubble dynamics near moderately elastic CWs.

1. Introduction

Cavitation bubbles near compliant walls (CWs) [1] are widely encountered in natural processes and engineering applications, e.g., ultrasound detection [2], tissue injury [3–6], and medical therapy [7–9]. An in-depth understanding of the fundamental mechanisms behind bubble–CW interactions is the prerequisite to efficiently developing and utilizing such advanced technology. Because of its superior ability to treat non-linear non-equilibrium dynamics over theoretical and experimental approaches, numerical simulation has been widely used to investigate bubble–CW interactions in recent years [10–12]. A large amount of useful results have been obtained through numerical simulation, notably elevating our knowledge about bubble–CW interactions.

In the numerical simulation of bubble–CW interactions, the mathematical model of CWs [13–15] plays a critical role in the accurate prediction of the bubble dynamics. A comprehensive literature review has revealed that there are two mainstream mathematical models for representing CWs within the numerical simulation of bubble–wall interactions. In one model, the CW is viewed as an elastic fluid with finite or infinite thickness. The hydrodynamics of the elastic liquid are modeled using the potential flow assumption, and the motion of the CW is tracked by monitoring the liquid–liquid interface [16–27]. Although this model is relatively easy to implement and has been widely employed, Klaseboer et al. [28] have noted that the potential flow assumption is only valid for weakly elastic fluids, which undoubtedly restricts its further application. Furthermore, in many situations, the CW is ultrathin, and it is still very difficult, if even possible, for the elastic fluid model to simulate an ultrathin CW, such as a bio-membrane.

In the other model, the CW is modeled by general structural dynamics, which can be dated back to the original work carried out by Duncan and Zhang [29]. Hereafter, this original work is referred as the reference study. In the reference study, the CW was modeled as a spring-backed membrane, in which four properties, i.e., the mass per unit area m, membrane tension T, spring stiffness per unit area K, and radius R_{CW}, were used to represent its physical characteristics. Compared to the elastic fluid model, this model is able to treat a CW of any elasticity and thickness. Therefore, this model has been followed by numerous studies [30–48], through which a wider range of CW properties than the elastic fluid model has been involved.

After a careful check of the results of the reference study, it was found that the bubble became almost spherically compressed under the condition of moderately elastic CWs, which deviated significantly from the experimental observations [49,50]. It is well known that the deviation from the spherical symmetry increases proportionally to $R^{-0.25}$ where R is the instantaneous radius of the bubble [51]. Thus, a basic but important issue is raised from such inconsistency between the results of the reference study and the well-known theory and experimental observations. What

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are the reasons for such a deviation? Is the spring-backed membrane approach not valid for modeling CWs? As presented in the above discussion, the answer to this question is crucial to the validity and future development of the spring-backed membrane model for CWs. Because of the extensive application of the spring-backed membrane model and its variations to the computer simulation of bubble–CW interactions so far, we incline to believe that this model is capable of describing CWs. Therefore, specifically, if the spring-backed membrane model is assumed to be valid for modeling CWs, why did the reference study fail to reproduce the bubble shape evolution under the condition of moderately elastic CWs? In the reference study, the bubble was directly created from its maximum-volume stage, from which only the collapse phase can be modeled. However, most experimental and numerical studies investigate the bubble dynamics from its physical inception, in which the bubble is at its minimum-volume state. It is widely accepted that the bubble growth phase significantly affects the behavior of its subsequent collapse phase. Thus, modeling the bubble collapse phase independently of its foregoing growth phase is suspected to be partially responsible for the deviation. In the following, this is referred as Suspicion I. Furthermore, in the reference study, the initial liquid pressure on the CW was artificially set to be uniform. However, this is contrary to common sense; because of the heterogeneous pulsation of the bubble on the CW, the liquid pressure on the CW should not be uniform. Thus, the second suspicion (Suspicion II) is that the improper assignment of the initial liquid pressure on the CW is the other source of the deviation.

This work is designed to check the validity of using the spring-backed membrane model for CWs within the numerical simulation of bubble–wall interactions. If the spring-backed membrane model is proven to be valid for modeling CWs, an explanation of why the reference study failed to reproduce the bubble behavior near a moderately elastic CW is proposed based on the two suspicions described above. The same numerical formulations and methods as were used in the reference study were chosen in our work. The bubble was created from its minimum-volume state, and its growth and collapse phases were described.

2. Numerical models and simulation setup

2.1. Numerical models

The numerical models employed in this work are exactly the same as those in the reference study. A brief description of the numerical models for the inviscid liquid and CWs is presented in the following. Detailed information about the model derivation and solving procedure can be found in the reference study.

Because of the very high bubble Reynolds number (on the order of 10^2–10^3), the intermediate incompressible liquid was modeled inviscid, using the Euler equation:

$$\begin{aligned} &\nabla \cdot \boldsymbol{u} = 0 \\ &\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) = -\frac{\nabla p}{\rho}, \end{aligned} \tag{1}$$

where ρ and $\boldsymbol{u}$ are the density and velocity vector of the liquid, respectively. p is the static pressure of the liquid. Eq. (1) can be further be reformulated using the potential flow assumption as

$$\begin{aligned} &\nabla \cdot \nabla \phi = 0 \quad \textit{where} \quad \boldsymbol{u} = \nabla \phi \\ &\rho \frac{d\phi}{dt} = P_{ref} - p + \frac{1}{2}\rho |\nabla \phi|^2, \end{aligned} \tag{2}$$

where p_{ref} is the reference pressure in the liquid far from the bubble, which is equal to atmospheric pressure in this work. Following our previous work [52], Eq. (2) was solved by the boundary integral method (BIM) as

$$c(\boldsymbol{x})\phi(\boldsymbol{x}) + \int_S \phi(\boldsymbol{y}) \frac{\partial G(\boldsymbol{y},\boldsymbol{x})}{\partial n} dS = \int_S G(\boldsymbol{y},\boldsymbol{x}) \frac{\partial \phi(\boldsymbol{y})}{\partial n} dS, \tag{3}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ are discretized points on the boundary surface of the liquid field S. n is the normal inward vector of S. In this study, S contains the bubble surface and the interface between the liquid and the wall. As BIM is only applied to S instead of the interior of the liquid, $c(\boldsymbol{x})$ equals 2π. The Green function $G(\boldsymbol{y}, \boldsymbol{x})$ reads

$$G(\boldsymbol{y},\boldsymbol{x}) = \frac{1}{|\boldsymbol{y} - \boldsymbol{x}|}. \tag{4}$$

In the spring-backed membrane model, the movement of CW is expressed as

$$m\frac{\partial^2 \eta(r,t)}{\partial t^2} = T\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \eta(r,t)}{\partial r}\right) - K\eta(r,t) - (p_m(r,t) - p_{ref}), \tag{5}$$

where $\eta(r, t)$ is the vertical displacement of the CW, r is the radial position on the CW, and t is the time. T represents the CW tension. K is the spring stiffness per unit area. m is the mass per unit area of the CW. The pressure imposed by the liquid is $p_m(r, t)$. Eq. (5) was solved using the finite difference method with high-order discretization schemes.

The coupling between the liquid and CW is realized by the linearized equations for pressure and velocity:

$$\begin{aligned} &\frac{\partial \eta(r,t)}{\partial t} = \left.\frac{\partial \phi}{\partial z}\right|_{z=0} \\ &p_m(r,t) = -\rho\frac{\partial \phi}{\partial t} + p_{ref}. \end{aligned} \tag{6}$$

2.2. Simulation layout

The simulation layout in this work was also set to be identical to that in the reference study, as shown in Fig. 1. A cylindrical coordinate system was established with the symmetric axis z piercing the center of the created bubble. The radial axis r is perpendicular to z, where the origin is defined as the intersection of these two axes. The radius of CW is R_{CW}, beyond which is a rigid wall. An axisymmetric adiabatic bubble containing a non-condensable ideal gas was initially created at $z = Z_0$. Thus, the instantaneous gas pressure can be calculated as

$$p = p_{ref}\varepsilon(R_0/R)^{3\gamma} \quad \textit{with} \quad \varepsilon = p_0/p_{ref}, \tag{7}$$

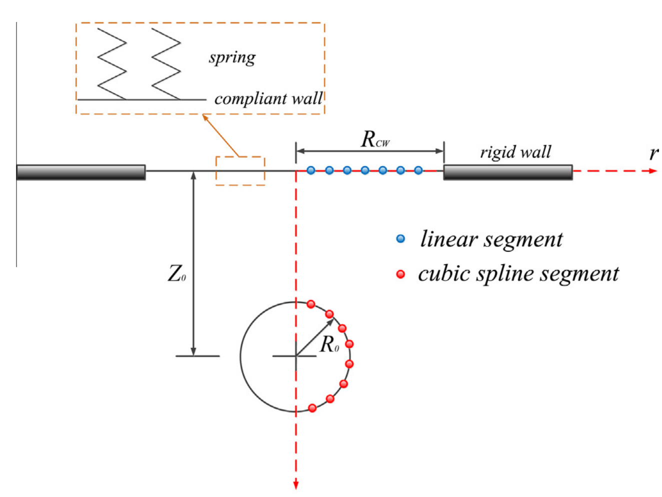

Fig. 1. Simulation layout and the discretization of the domain.