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# Shock response of a two-fluid cylindrical shell system containing a rigid core

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ABSTRACT

A submerged fluid-filled cylindrical shell containing a rigid co-axial core and subjected to an external shock wave is considered, and the fluid dynamics of such interaction is analyzed for the most general scenario of two different fluids. It is demonstrated that the phenomenology of the interaction in this case is fundamentally different from the case when the fluids are identical. In the latter case, all the most important wave propagation, reflection and focusing phenomena in the internal fluid that are observed for the shell without a core are also present when a core is placed inside the fluid, unless the core directly occupies the region of the fluid where the phenomena occur. When the fluids are different, however, it is possible that some phenomena are not observed even when the core does not occupy the respective region of the fluid. Due to the very high pressure that is often associated with the phenomena in question, this observation is of considerable practical significance in that it suggests the possibility of a very significant reduction of the peak pressure in the system by means of placing an additional structure inside the primary shell. The observations made are quantified using a number of pressure time-histories aimed at facilitating the pre-design analysis of shock-subjected fluid-interacting structures.

## 1. Introduction

Thin-walled cylindrical structures that are both submerged into and filled with fluid are common in naval architecture and ocean engineering, with underwater pipelines and ballast and fuel tanks of submersible vehicles being the most obvious examples. Although often the internal and external fluids are identical (e.g., ballast tanks), in many cases they are different (e.g., pipelines and fuel tanks), thus the results obtained under the assumption of the identical properties of the fluids are not always fully applicable. Furthermore, often marine systems possess complexity that is higher than that of a single fluid-interacting shell (ballast and fuel tanks, once again, being the most obvious example), and in such cases the relatively simple single-shell model is not fully adequate for even preliminary analysis. Thus, there are instances where it is critical to take into account the fact that the internal and external fluids interacting with the system at hand are different, but also the fact that the system has the structural complexity that is different, often significantly, from that of a single shell. At the same time, the studies that address the structural shock response for such relatively complex scenarios are rather scarce.

Specifically, there exist a number of studies where the most typical thin-walled structures that have fluid inside and outside are considered, with solutions often obtained under the assumption that the fluids can have different properties. To that end, the shock response of two concentric spherical shells was considered in [1], and two co-axial cylindrical shells were considered in [2], and although the solutions were obtained for the general case of different fluids, the results were only presented for the water–water case. A submerged fluid-filled spherical shell was considered in [3,4] and the solution introduced allowed for the consideration of two different fluids, but, again, the results were only presented for the case of identical fluids. The fluid dynamics of the two-fluid contact was also addressed experimentally, e.g. [5], but it appears that only the loading by identical internal and external fluids was considered.

A submerged fluid-filled shell was also addressed in [6] where both the scenario of the identical fluids and that of different ones were addressed in some detail. It was observed that when the fluids are different, the entire sequence of the shock wave propagation, reflection, and focusing phenomena changes dramatically if the difference in the fluid properties is significant enough. Namely,

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it was shown that four qualitatively different scenarios are possible depending of the properties of the fluids, with one of the scenarios being so different that even the classical [7,8] 'reflection–focusing' sequence of events in the internal fluid is no longer the case, and is replaced with a more complex 'focusing–reflection–focusing' sequence. Those observations lead us to believe that when the structural complexity of the system is increased, e.g. by means of adding a rigid core considered in the present study, the result will be an even more complex and interesting fluid dynamics of the interaction, and addressing such a possibility is one of the primary motivations for this work.

As for the studies of structurally complex shell systems, we mention here several representative studies without attempting a comprehensive review of the available literature. To that end, two rigid cylinders placed inside a channel conveying a shock wave were considered in [9], with images of the diffracted pattern developing around the cylinders presented. The shock response of three cylinders placed inside a larger cylindrical confinement was addressed in [10] with the focus on the complex scattering of the internal shock wave on the cylinders. The acoustic scattering on a system consisting of a sphere and a circular cylinder was addressed in [11] but only for the case of a stationary incident loading. A set of cylinders equally spaced inside the fluid layer confined by two much larger co-axial cylindrical structures was considered in [12], with the focus on the accurate modeling of the complex fluid–structure interaction effects in the system.

A system consisting of a cylindrical shell with a rigid co-axial core added to it was considered in [13], but only the case of identical internal and external fluids was considered. At the same time, it is expected that when the fluids are different, the resulting fluid dynamics of the interaction will be considerably more complex and interesting, and this expectation was another primary motivation for undertaking the present study.

Thus, summarizing the presented literature overview, we state that it appears that, despite the fact that structurally complex shell systems incorporating two different fluids are quite common in naval architecture and ocean engineering, there are no studies available that address shock response of such systems, and the goal of the present work is to fill this gap. While doing so, we will also provide a computationally efficient methodology suitable for use during the pre-design stage for extensive parametric studies, a tool that always seems to be well-received by the practitioner. Finally, the main theoretical outcome of this study will be the analysis of how combining the structural complexity (an added co-axial core) with the material one (two different fluids) affects the phenomenology of the fluid dynamics of the interaction.

## 2. Mathematical formulation

We consider a circular cylindrical shell of radius $r_0$ and thickness $h_0$, Fig. 1. We assume that $h_0/r_0 \ll 1$ and that the deflections of the shell surface are small compared to its thickness, and also assume that the Kirchhoff–Love hypothesis holds true. The density, Poisson's ratio, and Young's modulus of the shell material are $\rho_s$, $\nu$, and $E_s$, respectively, and the sound speed in the shell is $c_s$. The transverse and normal displacements of the middle surface of the shell are $v^*$ and $w^*$, respectively. The shell is submerged into fluid with the density $\rho_e$ and sound speed $c_e$ and is filled with fluid with the density $\rho_i$ and sound speed $c_i$. The fluids are assumed to be irrotational, inviscid, and linearly compressible. The shell is assumed to contain an absolutely rigid co-axial core of radius $a$. The polar coordinates $(\varrho, \theta)$ based on the axis of the shell are employed.

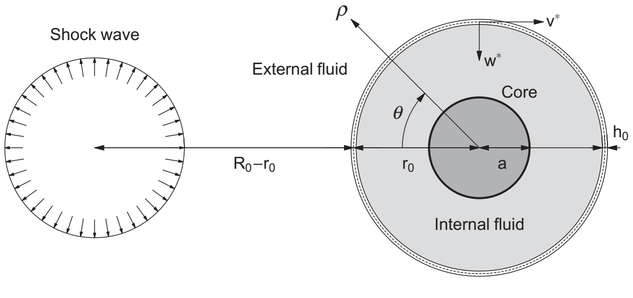

**Fig. 1.** Schematic of the problem.

We assume that the shell is subjected to an external shock wave with the fluid velocity potential $\phi_0$ in it given by

$$\phi_0 = -\frac{\lambda p_\alpha S_R}{\rho_e R^*} \mathrm{e}^{-(\tau - c_e^{-1}(R^* - S_R))\lambda^{-1}} \mathrm{H}(\tau - c_e^{-1}(R^* - S_R)), \tag{1}$$

where

$$R^* = \sqrt{R_0^2 + \varrho^2 - 2R_0\varrho\cos\theta}, \tag{2}$$

$p_\alpha$ is the pressure in the front of the wave when it first contacts the shell, $\lambda$ is the rate of exponential decay, $S_R = R_0 - r_0$ is the distance between the source and the shell (the shock wave's stand-off), and H is the Heaviside unit step function.

We note that such a formulation implies two simplifications of the fully-elastic, fully three-dimensional representation of a shell interacting with a shock wave. The first one is the use of the Kirchhoff–Love hypothesis which implies that cross sections initially plane and perpendicular to the middle surface of the shell remain so after deformation, and effectively means that the transverse shear deformations are disregarded. The validity of the use of the Kirchhoff–Love hypothesis has been addressed in some detail in our earlier work [16,17], and we have shown that in order to accurately reproduce the complex structure of the hydrodynamic field radiated by a shell in response to a shock or impulse loading, it is necessary to either model the shell as an elastic three-dimensional body [16,18], or employ a more advanced shell theory such as, for example, the Reissner–Mindlin theory [19–21,17]. The latter approach was found to be more computationally attractive since, unlike a fully three-dimensional model, it allows one to avoid abandoning all simplifying assumptions that are possible due to the unique geometry of a shell. It was observed, however, that when the shell is thin enough ($h_0/r_0 \leqslant 0.01$), the difference between the hydrodynamic fields produced by the Kirchhoff–Love model and more advanced models is limited to a few very high-frequency, highly localized wave formations that have little effect

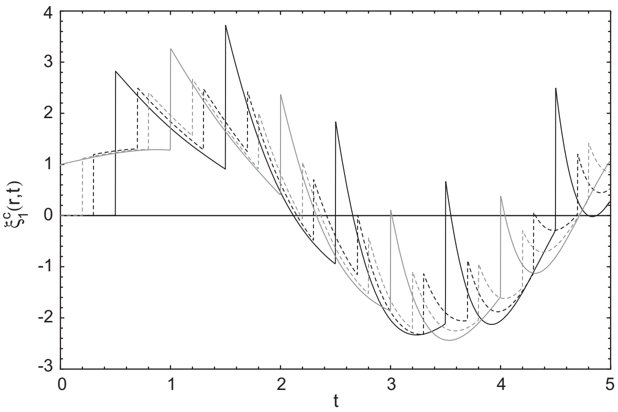

**Fig. 2.** Internal response function $\xi_1^c(r,t)$ for $a = 0.50$ and various $r$: $r = 0.50$, solid black line; $r = 0.70$, dashed black line; $r = 0.80$, dashed grey line; and $r = 1.00$, solid grey line.