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Numerical investigation of instability patterns and nonlinear buoyant exchange flow between enclosures by variable density approach

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ABSTRACT

The buoyancy driven flow characteristics through horizontal passage between two enclosures are numerically investigated. The two-dimensional physical model consists of upper and lower enclosures filled with cold and hot fluids connected through ceiling vent. Non-Boussinesq variable density approach is used to model the density variations by primitive variable method. The governing equations are solved by Simplified Marker and Cell (SMAC) algorithm on non-staggered grid using high accuracy compact finite difference schemes. The Grashof number is varied from $Gr = 10^6$ to 5×10^7. The nonlinear exchange of lighter and heavier fluids through vent are investigated by varying vent aspect ratio. The net mass flow rate through horizontal passage are oscillatory and bidirectional. The critical Grashof number is identified, and beyond this instabilities intensifies leading to complex flow behavior inside enclosures. The vent widths $D = 0.05H$ and $0.2H$ reduces flow perturbations and enhances stable flow behavior across the vent. Chaotic flow originates for critical vent widths $0.1H \leqslant D \leqslant 0.15H$, and nonlinear oscillations evolves till the system reaches quasi-steady state. Reduced vent thickness results in higher oscillation frequencies and better mixing rates between enclosures. The present mathematical model and numerical method showed good agreement with the existing results available in literature.

1. Introduction

The buoyancy induced flow through enclosure openings has wide applications in natural ventilation, fire dynamics, solar collectors and cooling of electronic applications. Most of the classical problems [1–3] on natural convection flow phenomena were modeled with Boussinesq approximation. The approximation is valid when product of thermal expansion coefficient and temperature difference is insignificant ($\beta\Delta T \ll 1$). However, for applications with larger temperature gradients, it is inappropriate to evaluate density variations by Boussinesq approximation. The heat transfer by natural convection in cavities [4–7] with differentially heated side walls are widely investigated in literature. Numerical simulations were performed in square cavity [4] for wide range of Rayleigh numbers $Ra = 10^3–10^{16}$. The above well known benchmark numerical solutions provide insight into the free convection transport phenomena. Numerical study was performed in thermally driven square cavity [8] in non-Boussinesq regime by solving compressible Navier–Stokes equation. The convective term was evaluated by explicit third order discretization scheme and stiffness was treated by preconditioning technique. A similar numerical study was carried out by compressible approach using finite volume method [9] to incorporate larger density variations.

The Gay-Lussac (Ga) number quantifies density variations of the working fluid [10,11] in variable density formulations. The Boussinesq approximation is valid when Gay-Lussac's number is negligible, close to zero. The effects of Ga on the flow characteristics was investigated [11] for fixed Rayleigh and Prandtl numbers. They identified that velocity fields were significantly affected by increasing Gay-Lussac's number. Recent numerical study on high thermo-buoyant flows [12] in square cavity with bottom heat source has compared the heat transfer characteristics between incompressible Boussinesq and compressible flows. Similar numerical investigations were carried out to predict the conjugate natural convection flows in vertical annulus [13] and capability of Boussinesq and non-Boussinesq models were discussed.

All the above mentioned studies are related to natural convection in cavities which are heated from the left and bottom boundaries. However numerical studies on buoyancy induced mixing through horizontal vent between two compartments filled with heavier and lighter fluids are limited. Similar computational studies in past [14–16] are within the frame work of Boussinesq approximation. The flow characteristics through vents are oscillatory and bidirectional. The transport phenomena between enclosures were classified based on four modes [15] namely diagonal exchange

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counter modes, counter flow exchange mode, standing wave mode and trapped-vortex mode. The effects of vent aspect ratio [16] on bidirectional flows through horizontal vents were investigated to determine the frequency of oscillatory flow pattern. The buoyancy driven unsteady mixing between hot and cold fluids leads to the formation of thermal plumes and is well-known in literature as Rayleigh Taylor instability problems [17–19].

Analytical and numerical investigations [20–22] were performed to study the entrainment effects due to buoyancy induced air flow through horizontal vent in enclosures. It was identified that mass flow rates through the ceiling vents [22] are significantly affected by varying the heat source and vent locations. Higher order numerical schemes [23–25] were proposed in literature to evaluate the convective terms in Navier–Stokes equation to yield high accuracy solutions. The compact finite difference schemes were developed to determine unsteady numerical solutions accurately and it was found to be computationally efficient. The high accuracy compact schemes was applied for mixed convection [26] and natural convection [22,27] problems where the density variations are modeled by Boussinesq approximation. However implementation of compact schemes in variable density problems suitable for low Mach number flows or weakly compressible flows are limited.

In literature, numerical studies on the entrainment effects through horizontal vent modeled by variable density approach are limited. This has been the motivation for the present investigation. In the present article buoyancy induced mixing process between two square enclosures connected through central horizontal passage is numerically investigated. The lower and upper enclosures are filled with hot and cold fluids. A non-Boussinesq variable density approach is implemented to evaluate the density variations. The results are presented by varying different Grashof numbers and the critical Grashof number above which flow instabilities develop across the vent are determined. The effects of vent aspect ratio on flow characteristics are studied by varying vent width and thickness.

2. Governing equations and boundary conditions

The lower and upper enclosures are filled with hot (T_h) and cold fluids (T_c) as shown in Fig. 1. Horizontal vent of width D and thickness H_v facilitates the buoyancy induced mixing process. The natural convection flow is modeled as unsteady, two-dimensional, compressible flow problem suitable for low Mach number flows. The thermo-physical properties of fluid are constant, except the change in density which are evaluated from ideal gas law. The governing equations for conservation of mass, momentum and energy are as follows:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}=-\frac{\partial(p-p_\infty)}{\partial x}+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\right)+\frac{\partial}{\partial y}\left(\mu\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]\right) \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}=-\frac{\partial(p-p_\infty)}{\partial y}+\frac{\partial}{\partial x}\left(\mu\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]\right)+\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\right)-(\rho-\rho_\infty)g \tag{3}$$

$$\frac{\partial(\rho T)}{\partial t}+\frac{\partial(\rho uT)}{\partial x}+\frac{\partial(\rho vT)}{\partial y}=\frac{k}{C_p}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{4}$$

The equation of state relates density with pressure and temperature and are as follows:

$$p=\rho RT \tag{5}$$

where ρ is the density of the fluid; μ is the dynamic viscosity; k is the thermal conductivity; g is the acceleration due to gravity. The pressure work and viscous dissipation terms in energy equation are ignored for low Mach number flows.

The modified continuity equation is as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}\right) \tag{6}$$

The time discretized momentum equations are given as follows:

$$u^{n+1}=u^*-\frac{\Delta t}{\rho}\frac{\partial p}{\partial x} \tag{7}$$

$$v^{n+1}=v^*-\frac{\Delta t}{\rho}\frac{\partial p}{\partial y} \tag{8}$$

$$u^*=u^n+\Delta t\left[\frac{1}{\rho}\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\right)+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]\right)-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}\right] \tag{9}$$

$$v^*=v^n+\Delta t\left[\frac{1}{\rho}\frac{\partial}{\partial x}\left(\mu\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]\right)+\frac{1}{\rho}\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\right)-\frac{(\rho-\rho_\infty)g}{\rho}-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}\right] \tag{10}$$

The pressure Poisson equation given below is obtained by substituting Eq. (7 and 8) into L.H.S. of eqn (6).

$$\nabla^2 p=\frac{\rho}{\Delta t}\left[\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}\right)\right] \tag{11}$$

where u^* and v^* are the predicted velocity fields obtained from Eqs. (9 and 10) and ρ, u, v are the density and velocity fields at time t^n.

The following non-dimensional variables are used to obtain the dimensionless governing equations:

$$X=\frac{x}{H};\ Y=\frac{y}{H};\ U=\frac{u}{V_c};\ V=\frac{v}{V_c};\ \tau=\frac{tV_c}{H};\ \theta=\frac{T-T_c}{T_h-T_c};$$
$$V_c=(g\beta\Delta TH)^{\frac{1}{2}};\ P=\frac{p-p_\infty}{\rho_\infty V_c^2};\ \varrho=\frac{\rho}{\rho_\infty}$$

The dimensionless governing equations are as follows:

$$\frac{\partial \varrho}{\partial \tau}+\frac{\partial(\varrho U)}{\partial X}+\frac{\partial(\varrho V)}{\partial Y}=0 \tag{12}$$

$$\frac{\partial(\varrho U)}{\partial \tau}+\frac{\partial(\varrho UU)}{\partial X}+\frac{\partial(\varrho UV)}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\partial}{\partial X}\left(\frac{2}{(Gr)^{\frac{1}{2}}}\frac{\partial U}{\partial X}-\frac{2}{3(Gr)^{\frac{1}{2}}}\left[\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right]\right)+\frac{\partial}{\partial Y}\left(\frac{1}{(Gr)^{\frac{1}{2}}}\left[\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right]\right) \tag{13}$$

$$\frac{\partial(\rho V)}{\partial \tau}+\frac{\partial(\varrho UV)}{\partial X}+\frac{\partial(\varrho VV)}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\partial}{\partial X}\left(\frac{1}{(Gr)^{\frac{1}{2}}}\left[\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right]\right)+\frac{\partial}{\partial Y}\left(\frac{2}{(Gr)^{\frac{1}{2}}}\frac{\partial V}{\partial Y}-\frac{2}{3(Gr)^{\frac{1}{2}}}\left[\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right]\right)-\frac{(\varrho-1)}{Ga} \tag{14}$$

$$\frac{\partial(\varrho\theta)}{\partial \tau}+\frac{\partial(\varrho U\theta)}{\partial X}+\frac{\partial(\varrho v\theta)}{\partial Y}=\frac{1}{Pr(Gr)^{\frac{1}{2}}}\left[\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right] \tag{15}$$

where $Gr=\frac{g\beta\Delta TH^3}{\nu^2}$ is the Grashof number; $Pr=\frac{\nu}{\alpha}$ is the Prandtl number; $Ga=\beta\Delta T$ is the Gay-Lussac's number.

The solid walls are treated with adiabatic boundary conditions and no-slip boundary conditions are specified for velocity fields. At initial time $\tau=0$, the lower enclosure and lower half of the horizontal passage are filled with hot fluid ($\theta=1$), meanwhile the horizontal passage upper half and upper enclosure are filled with cold fluid ($\theta=0$).

$$at\ \tau=0: U=V=0 \tag{16}$$