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A spectral-element discontinuous Galerkin lattice Boltzmann method for simulating natural convection heat transfer in a horizontal concentric annulus

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ABSTRACT

We present a spectral-element discontinuous Galerkin lattice Boltzmann method to solve incompressible natural convection flows based on the Bousinessq approximation. A passive-scalar thermal lattice Boltzmann model is used to resolve flows for variable Prandtl number. In our model, we solve the lattice Boltzmann equation for the velocity field and the advection–diffusion equation for the temperature field. As a result, we reduce the degrees of freedom when compared with the passive-scalar double-distribution model, which requires the solution of several equations to resolve the temperature field. Our numerical solution is represented by the tensor product basis of the one-dimensional Legendre–Lagrange interpolation polynomials. A high-order discretization is employed on body-conforming hexahedral elements with Gauss–Lobatto–Legendre quadrature nodes. Within the discontinuous Galerkin framework, we weakly impose boundary and element-interface conditions through the numerical flux. A fourth-order Runge–Kutta scheme is used for time integration with no additional cost for mass matrix inversion due to fully diagonal mass matrices. We study natural convection fluid flows in a square cavity and a horizontal concentric annulus for Rayleigh numbers in the range of $Ra = 10^3$–10^8. We validate our numerical approach by comparing it with finite-difference, finite-volume, multiple-relaxation-time lattice Boltzmann, and spectral-element methods. Our computational results show good agreement in temperature profiles and Nusselt numbers using relatively coarse resolutions.

1. Introduction

Natural convection flow simulations have been an active area of research for many years. These flows are set in motion by a buoyancy force that occurs as a result of a small density gradient and the presence of an external force such as gravity. Understanding the behavior of natural convection flows is important in nuclear reactor design, cooling of electronic equipment, and determination of heat loss from steam pipes.

In recent decades, thermal lattice Boltzmann methods (TLBMs) have emerged as reliable methods for simulating natural convection flows. TLBMs generally fall into two approaches: the multispeed approach and the passive-scalar approach. The multispeed approach is an extension of the isothermal model, where the density distribution function is solely used to describe the mass, momentum, and temperature [1,2]. The passive-scalar approach uses additional equations, independent of the density distribution, to describe the temperature. When viscous heating and compression work due to pressure are negligible, as is the case in most natural convection flows, the temperature does not influence the momentum—it is advected and diffused “passively” [3].

The multispeed approach does have limitations. In particular, it suffers from severe numerical instability and restricts the Prandtl (Pr) number to a fixed quantity [1]. However, numerous models have been proposed to rectify these issues. In [4], McNamara et al. were able to improve the stability by implementing a Lax–Wendroff advection scheme. Using higher-order symmetric velocity lattices, Vahala et al. [5] showed better stability properties over lower-order symmetric lattices. Prasianakis and Karlin [6] built a model using the standard velocity lattice (D2Q9), which incorporated equilibrium expansions up to the fourth order in velocity and correction terms to the lattice Boltzmann equation (LBE) in order to enhance stability for high Rayleigh number (Ra) flow. The correction terms also allowed their model to investigate

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variable Pr. Watari and Tsutahara [7] proposed a finite-difference lattice Boltzmann method (FDLBM) that utilized a second-order upwinding difference scheme to improve stability. And to investigate variable Pr, Soe et al. [8] introduced an extended collision matrix without affecting the stability.

One version of the passive-scalar approach utilizes a double-distribution model based on the multiple component LBE proposed by Shan and Chen [3]. In this approach, one component (i.e., density distribution function) represents motion of the fluid and the other (i.e., energy distribution function) describes the passive temperature field. Two independent relaxation times are utilized for each component, thus allowing for variable Pr. In [9], Shan showed that the double-distribution model enhanced numerical stability over the multispeed approach for high Ra. He et al. [10] also proposed a double-distribution model in which the density distribution function recovers the macroscopic mass and momentum variables while an internal energy density distribution function recovers the energy. Because the model in [10] directly solves evolution of the internal energy, a Chapman–Enskog multiscale expansion analysis shows that viscous heat dissipation and compression work are correctly recovered in the macroscopic energy equation.

Since the work of He et al. [10], simpler double-distribution models have been proposed in the incompressible limit. Both Palmer and Rector [11] and Peng et al. [12] neglected viscous dissipation entirely and dropped complicated spatial gradients to study Rayleigh-Bénard convection and natural convection within a square cavity. In [13], Shi et al. proposed a double-distribution model that incorporates only viscous heat dissipation to study thermal Couette flow. Guo et al. [14] proposed a double-distribution model based on the total energy, which allows for a simpler computation of viscous dissipation and compression work. Others have proposed smaller lattice velocity models for the energy distribution functions [15].

The double-distribution model has also been used on irregular or unstructured grids to handle natural convection flows. Dixit and Babu [16] employed an interpolation supplemented lattice Boltzmann method [17] on a nonuniform mesh to study natural convection in a square at $Ra > 10^6$. Shi et al. [18] extended the method proposed by Guo and Zhao [19] and used FDLBM on the polar representation of the double-distribution model. Shu et al. [20] used a Taylor series expansion and least-squares-based lattice Boltzmann method (TLLBM) to solve the double-distribution model. The TLLBM has proved useful for complex geometries [21]. Finite-volume lattice Boltzmann methods (FVLBMs) have also been proposed and implemented on unstructured meshes [22]. Although FVLBM has been applied to isothermal flows, an extension to either a multispeed or double-distribution model seems feasible.

Another passive-scalar approach is to solve the macroscopic energy equation for the temperature and couple it with the isothermal LBE in order to resolve the velocity. This approach is beneficial for flows with negligible viscous dissipation, and therefore the macroscopic energy equation simplifies to an advection–diffusion equation for the temperature. This model eliminates the need to solve multiple equations as is required in the double-distribution model. In addition, flows with variable Pr number can be investigated. Lallemand and Luo [23] proposed this type of approach, solving the advection–diffusion equation for the temperature using a finite-difference method. They showed enhanced stability for simple Cartesian geometries such as a cubic box. For complex geometries, however, finite-difference stencils may not have the same symmetries as the underlying discrete velocity, and extrapolation might cause loss of local conservation.

Implementation of physically accurate hydrodynamic and thermal boundary conditions is crucial in both the multispeed and passive-scalar models. Extensive research on boundary treatment techniques has been done and we refer the reader to the following literature: [24–30].

In this paper, we present a spectral-element discontinuous Galerkin (SEDG) method to solve a passive-scalar thermal lattice Boltzmann model. Our numerical scheme is extended from the previously developed spectral-element discontinuous Galerkin lattice Boltzmann method (SEDG-LBM) presented in [31]. We include a force term, resulting from the Bousinessq approximation [9], into the discrete Boltzmann (DB) and lattice Boltzmann (LB) equations. This approach allows us to examine flows in the incompressible limit (i.e. for low Mach (Ma) numbers and small density fluctuations).

We use the SEDG-LBM to solve the LBE for the density distribution function thereby resolving the mass and momentum conservation laws. With proper coupling to the LBE, we then determine the temperature field by solving the advection–diffusion (i.e. energy) equation. We use a high-order spectral-element discontinuous Galerkin (SEDG) discretization based on the tensor product basis of the one-dimensional Legendre–Lagrange interpolation polynomials. Our SEDG discretization is employed upon body-conforming hexahedral elements with Gauss–Lobatto–Legendre (GLL) grid points. Bounceback boundary conditions are applied weakly through the numerical flux without the additional effort of interpolation for complex geometries as required by other lattice Boltzmann (LB) schemes [25–27].

The paper is organized as follows. In Section 2, we present the governing equations, namely, the LBE with a Bousinessq approximation and the advection–diffusion equation. In Section 3, we discuss the formulation of our numerical scheme. Section 4 presents computational results and their validation for natural convection heat transfer in a square cavity and horizontal concentric annulus. We discuss our conclusions in Section 5.

2. Governing equations

In this section we describe our governing equations for natural convection flows. We derive the lattice Boltzmann equation with a forcing term and the formulation for the collision and streaming steps. We also present a simplified macroscopic energy equation for incompressible natural convection flows.

2.1. Lattice Boltzmann equation: Collision and Streaming

We write the discrete Boltzmann equation with a forcing term, where the collision term is approximated by the Bhatnagar–Gross–Krook, or single-relaxation-time, operator [32]:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{e}_\alpha \cdot \nabla f_\alpha = -\frac{f_\alpha - f_\alpha^{eq}}{\lambda} + \frac{(\mathbf{e}_\alpha - \mathbf{u}) \cdot \mathbf{G} f_\alpha^{eq}}{\rho c_s^2}, \tag{1}$$

where f_α $(\alpha = 0, 1, \ldots, N_\alpha)$ is the particle density distribution function defined in the direction of the microscopic velocity $\mathbf{e}_\alpha$, λ is the relaxation time, and N_α is the number of microscopic velocities. We consider the two-dimensional 9-velocity model (D2Q9) associated with $\mathbf{e}_\alpha = (0,0)$ for $\alpha = 0$; $\mathbf{e}_\alpha = (\cos\theta_\alpha, \sin\theta_\alpha)$ with θ_α=$(\alpha-1)\pi/2$ for α = 1, 2, 3, 4; and $\mathbf{e}_\alpha = \sqrt{2}(\cos\phi_\alpha, \sin\phi_\alpha)$ with ϕ_α=$(\alpha-5)\pi/2 + \pi/4$ for α = 5, 6, 7, 8. The second term on the right-hand side of Eq. (1) represents the force term. $\mathbf{G}$ is the external body force, depending on space and time. We consider a Bousinessq approximation for $\mathbf{G}$. Details on the formulation for $\mathbf{G}$ are discussed in Section 4. The equilibrium distribution function is given by

$$f_\alpha^{eq} = t_\alpha \rho \left[1 + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2c_s^2}\right], \tag{2}$$

where ρ is the density; $\mathbf{u}$ is the macroscopic velocity; $t_0 = 4/9, t_{\alpha=1,4} = 1/9$, and $t_{\alpha=5,8} = 1/36$ are the weights; and