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Hybrid aeroacoustic computation of a low Mach number non-isothermal shear layer

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ABSTRACT

Direct numerical simulation of noise generated by low speed flows requires strong numerical constraints related to the different scales in space and time for the dynamics of the flow and the propagation of sound waves. At low Mach numbers, the aeroacoustic hybrid approaches initiated by Hardin and Pope (1994) [5] based on separate calculations for the flow and for the acoustic radiation, are therefore attractive. In this paper, we show that such methods can be used for the general case of non-constant density or temperature. The starting point is an asymptotic expansion of the full Navier–Stokes equations that gives a set of equations that retain the presence of density and temperature inhomogeneities, allowing access to the dynamic quantities without the stability constraints related to acoustic waves. Then starting from the solutions of flow fluctuating quantities, we propose several possible developments of the equations to obtain the acoustic field. They lead to different sets of equations and source terms depending on the level of simplifying assumptions: the Perturbed Low Mach Number Approximation (**PLMNA**) or the linearized Euler equations (**LEE**) linearized with respect to the mean flow. An isothermal and a non-isothermal spatially evolving mixing layer are taken as test problems. The solutions of the proposed hybrid methods show a satisfactory behavior compared with the reference solution given by a compressible DNS.

1. Introduction

Computational aeroacoustics has established itself as a powerful tool to predict noise generated aerodynamically [1]. Two broad classes of methods are available. The first class performs direct noise computations, whereby both the near-field flow dynamics and the far-field sound radiation are computed simultaneously by solving the compressible Navier–Stokes equations (see for instance [2–4]). A clear advantage of this approach is that the connection between the flow dynamics and the sound produced occurs in a natural way and requires no model for the sound source. However, this method requires large computational resources and is particularly inefficient in the low Mach number range, due to stability restrictions. This has motivated the second class of methods, known as hybrid acoustic methods, such as the splitting technique developed by Hardin and Pope [5]. In hybrid methods, the calculation of the flow dynamics and that of the sound produced are performed in two different stages. The flow dynamics, calculated during the first stage, is used to calculate a source term that is passed on to the acoustic solver in the second stage. This strategy implies that there cannot be any feedback of the sound to the flow dynamics. This may not always be the case, as for example during the calculation of the sound produced by a diaphragm in a duct [6].

For Mach numbers less than about 0.3, when the flow dynamics are calculated using an incompressible solver, these methods can lead to a speed-up factor inversely proportional to the Mach number over the direct noise computation method [7]. Moreover, hybrid methods make it possible to compute the sound produced for a whole range of Mach numbers, based on a single incompressible flow solution. One drawback of hybrid methods is that they rely on a splitting of the variables which is partially arbitrary, meaning that the definition of the source term may also vary from one formulation to the other.

In hybrid methods, it is often assumed that the incompressible Navier–Stokes equations can be used to compute the flow [5,7–9]. While the dynamics are close to incompressible at low Mach numbers, the flow may have temperature inhomogeneities that cannot be handled in the strictly incompressible context (see Eq. (9) below). However, density and temperature inhomogeneities, when not of an acoustic nature, can be accounted for by using a low Mach number solver [10]. Such a Low Mach Number Approximation (**LMNA**) has a stability restriction which is equivalent to that of the incompressible solver; this is used in combustion problems [11,12]. It has also been used in acoustic hybrid methods [13–17]

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although few works apply the method to study the acoustic radiation of a non-isothermal flow: in Ref. [14], the flow obtained using the **LMNA** is used as input for a non-isothermal form of the Lighthill analogy to compute the sound radiated by a non-isothermal temporal mixing layer. In ref [17], the flow for a non-isothermal temporal mixing layer is once again calculated using the **LMNA** and the linearized Euler equations (**LEE**) are solved with a pressure gradient source term in the momentum equations to compute the sound. The source term is obtained by using an analogy. The acoustic results are successfully compared with that from a direct noise calculation, which shows the potential of the **LMNA** for non-isothermal flows.

The noise computation in hybrid methods is carried out by using perturbed equations together with corresponding source terms, which depend on the precise formulation chosen by their authors. In any case, the perturbed equations are connected to the linearized Euler equations [3,9,16,18]. A well known problem with the **LEE** is that they can sustain unstable vortical modes that can corrupt the noise computation. One possibility to avoid this is to work in the frequency domain[19]. In the time domain, one strategy is to modify the equations so that they no longer support the unstable mode [3,18,20]. This can be done, for example, by removing the term $v'dU/dy$ responsible for instability in a mean shear flow $U(y)$. However, a detrimental effect of this method is that some sound/flow interactions are neglected [21]. Yet another possibility is to keep the **LEE** unchanged, but to use a source term which reduces the vortical mode excitation (a rotational free source term) such as a pressure gradient in the momentum equation [16].

In the present paper, the objective is to develop a hybrid method for a low Mach number flow and to assess its use for predicting noise radiation by isothermal and non-isothermal mixing layers. The Low Mach Number Approximation (**LMNA**) flow solver is first presented in Section 2. The acoustic solver based on a perturbation of the compressible Navier–Stokes equations, the Perturbed Low Mach Number Approximation (**PLMNA**) is then presented in Section 3. The perturbation method used to obtain the acoustic solver differs from the analogy-based approach followed in [16,17] and is mathematically more rigorous. The **LEE**, linearized around a time-averaged flow field, are retrieved from the **PLMNA**, and the source term is discussed. Following the method of Seo and Moon [18], filtered versions of the **PLMNA** and **LEE** are presented in Section 4, which have reduced vorticity. A propagation equation based on the **LEE** is proposed in Section 5 to compare the present methods with Lilley's and Ribner's analogies. To validate the hybrid method, a spatially evolving mixing layer with or without temperature gradients and its sound radiation are computed in Section 6 using several source terms. A spatial mixing layer is a more demanding configuration than the temporal mixing layer used in [14,17] due to more stringent boundary conditions. The results are compared to those provided by direct noise computation [4], and conclusions are given in Section 7.

2. The Low Mach Number Approximation (LMNA)

The first part of the hybrid approach consists in calculating the evolution of the variables in the flow. In the present study, this is done by solving a low Mach number limit of the Navier–Stokes equations [11,12,15,16]. In this limit, the acoustic waves are filtered out so that they do not constrain the stability limit. However, temperature and density inhomogeneities are retained, which are necessary for dealing with non-isothermal flows. The equations are obtained from the full non-dimensionalized Navier–Stokes equations that read:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho e u_j}{\partial x_j} + p\frac{\partial u_j}{\partial x_j} = \frac{\tau_{ij}}{Re}\frac{\partial u_i}{\partial x_j} + \frac{(\gamma-1)^{-1}}{M^2 RePr}\frac{\partial}{\partial x_j}\left(\mu\frac{\partial T}{\partial x_j}\right) \quad (3)$$

$$p = \frac{\rho T}{\gamma M^2} \quad (4)$$

where ρ, u_i, p, T are the density, velocity, pressure, and temperature respectively. Re, M and Pr stand respectively for the Reynolds, Mach and Prandtl number, and γ is the ratio of specific heats at constant pressure and volume, $\mu = \mu(T) = \mu^*/\mu^*_{ref}$ is the normalized viscosity (calculated through the Sutherland law). The internal energy per unit volume ρe (for an ideal gas) and the viscous stress tensor τ_{ij} are as follows:

$$\rho e = \frac{p}{\gamma - 1}, \quad \tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right). \quad (5)$$

The normalization is done by introducing a length scale $L^*_{ref_h}$, a velocity scale $U^*_{ref_h}$, a time scale $t^*_{ref_h} = U^*_{ref_h}/L^*_{ref_h}$, a density scale $\rho^*_{ref_h}$, a pressure scale $p^*_{ref_h} = \rho^*_{ref_h} U^{*2}_{ref_h}$, and a temperature scale $T^*_{ref_h}$.

Then, it follows that $Re = \rho^*_{ref_h} U^*_{ref_h} L^*_{ref_h}/\mu^*_{ref_h}$, $M = U^*_{ref_h}/\sqrt{\gamma r^* T^*_{ref_h}}$ and $P_r = \mu^*_{ref_h} c^*_p/\lambda^*_{ref_h}$ with $\lambda^*_{ref_h}$ the thermal conductivity. The subscript $_h$ indicates a normalization for the hydrodynamic solver, and a different normalization will be encountered below for the acoustic solver.

For low Mach numbers (typically $M < 0.3$), the parameter $\epsilon = \gamma M^2$ is small and is used to conduct an asymptotic expansion of the equations. The variables are expanded according to:

$$\begin{aligned} &\rho = \rho_0 + \epsilon\rho_1 + \cdots, \quad u_i = u_{i_0} + \epsilon u_{i1} + \cdots \\ &T = T_0 + \epsilon T_1 + \cdots, \quad p = \tfrac{p_0}{\epsilon} + p_1 + \cdots. \end{aligned} \quad (6)$$

Introducing these expansions into Eqs. (1)–(4), an asympotic expansion of the Navier–Stokes equations is obtained. Keeping the lowest order terms in ϵ provides the set of **LMNA** equations as developed by McMurtry et al. [11]:

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho_0 u_{i_0}}{\partial x_i} = 0 \quad (7)$$

$$\frac{\partial \rho_0 u_{i_0}}{\partial t} + \frac{\partial \rho_0 u_{i_0} u_{j_0}}{\partial x_j} = -\frac{\partial p_1}{\partial x_i} + \frac{1}{Re}\frac{\partial \tau_{ij_0}}{\partial x_j} \quad (8)$$

$$\rho_0 \frac{\partial u_{i_0}}{\partial x_i} = \frac{1}{ReP_r T_0}\frac{\partial}{\partial x_j}\left(\mu\frac{\partial T_0}{\partial x_j}\right) \quad (9)$$

$$p_0 = \rho_0 T_0 \quad (10)$$

$$\frac{\partial p_0}{\partial t} = \frac{\partial p_0}{\partial x_i} = 0. \quad (11)$$

Note that Eq. (11) actually comes from the boundary condition for an unbounded domain.

Here the acoustic waves are filtered out [10] but density fluctuations that are not of an acoustic nature are retained. Thus, the stability of the **LMNA** solver is equivalent to that of an incompressible solver. In the present study, density fluctuations are caused by a gradient of temperature, where temperature and density are related by Eqs. (10) and (11). The **LMNA** equations requires solving a Poisson's equation that is slightly modified compared to the one solved for an incompressible case. The method presently used is described in [11,16], an alternative method being given in [22].

3. Models for solving the acoustic problem

The hydrodynamic quantities are calculated by solving the **LMNA** system. Then two different strategies are used to compute