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Asymptotic solution for a penny-shaped near-surface hydraulic fracture ☆

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Abstract

This paper considers the large time asymptotic behavior of a near-surface hydraulic fracture, that is, when the radius (R) is much larger than the depth (H). The fracture is analyzed as an elastically clamped circular plate and stress intensity factors are determined by matching the outer plate problem to the inner problem of a near-surface semi-infinite crack. In the zero-viscosity limit, we derive two terms of a large R/H asymptotic solution. Comparison shows that the accuracy of some published numerical results deteriorates for $R/H > 5$. This is corrected using smaller element size to ensure that the crack-tip element is entirely in the region that is well-approximated by a square-root tip asymptote.

Keywords: Hydraulic fracture; Blister crack; Fracture mechanics; Matched asymptotics; Free-surface effects

1. Introduction

Hydraulic fracturing in near-surface environments, used in quarrying operations over a century ago [1,2], has been a subject of increasing interest in recent years. Driving this interest are applications of hydraulic fracture for mine caving [3], environmental remediation [4], and for modelling certain geophysical processes [2,5,6]. In addition to experimental contributions [4,7], these applications have motivated recent work aimed at modelling near-surface hydraulic fractures [4,8].

Foundational to much of the recent work in hydraulic fracture modelling is the concept of regimes of propagation [9]. For example, if loss of fluid from the fracture into the solid matrix (fluid leak-off) is

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Nomenclature

B	a coefficient related to η and ν
C_2	local compliance, point load
D	plate flexural rigidity
E, E'	Young's, plane strain modulus
F, $\mathscr{F}$, $\mathscr{F}_k$, $\mathscr{F}_{\bar{k}}$	solution, scaled solution
G_{in} (G_{out})	energy release rate of inner (outer) problem
H	fracture depth (plate thickness)
K_{I}, K_{II}	mode I and II stress intensity factors (SIF's)
$K_{\text{Ic}}(K')$	critical mode I SIF, alternate ($K' = 4(2/\pi)^{1/2}K_{\text{Ic}}$)
K_M, K_N, K_T	stress intensity factor coefficients
L, L_k, $L_{\bar{k}}$	characteristic length (see Table 1)
M, N, T	moment, normal and shear force applied to inner problem
M_r	moment
P	point force at fracture center
p	internal pressure loading
Q	fluid injection rate
q	fluid flux
Q_r	shear force
R	fracture radius
r	radial coordinate
t, $\bar{t}$	time, characteristic time
w	fracture opening
α	a constant related to ν
β	a constant related to ν
δ	a constant associated with the energy release rate
ϵ, ϵ_k, $\epsilon_{\bar{k}}$	scaling parameter (see Table 1)
ϵ_1	scaling parameter, uniform pressure
ϵ_2	scaling parameter, point load
η	dimensionless clamping coefficient
γ, γ_k, $\gamma_{\bar{k}}$	scaled radius (see Table 1)
κ	elastic clamping coefficient
$\mu(\mu')$	fluid viscosity, alternate ($\mu' = 12\mu$)
ν	Poisson's ratio
Ω, Ω_k, $\Omega_{\bar{k}}$	scaled opening (see Table 1)
Ω_1	scaled opening, uniform pressure
Ω_2	scaled opening, point load
W, W_k, $W_{\bar{k}}$	characteristic opening (see Table 1)
Π, Π_k, $\Pi_{\bar{k}}$	scaled pressure (see Table 1)
ρ	scaled radial coordinate
σ	far-field normal stress parallel to free-surface
τ, $\tau_{\bar{k}}$	dimensionless time
$\mathscr{S}$, $\mathscr{S}_k$, $\mathscr{S}_{\bar{k}}$	scaling rule (see Table 1)
$\mathscr{G}_i$	dimensionless group
φ, $\mathscr{M}_k$, $\mathscr{M}_{\bar{k}}$	dimensionless viscosity (see Table 1)
$\mathscr{R}$	radius to depth ratio, R/H