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Application of network simulation method to viscous flows: The nanofluid heated lid cavity under pulsating flow

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ABSTRACT

In this paper, the Network Simulation Method (NSM) is used to model an unsteady, viscous, flow problem: the heated lid-driven filled with nanofluid and in the presence of a pulsating flow. Through this method is analyzed the influence of the amplitude, wave number and oscillation frequency of sinusoidal velocity waves at the lid on the convection performance of the cavity. It is stated that this method is simple and efficient for solving unsteady, viscous, Navier–Stokes equations, through the design and resolution of an electrical circuit network whose equations are formally equivalent to the ones of the fluid flow problem. Results show that, for the case of Pr = 3.93, Re = 50, Ri = 11.82, sinusoidal velocity waves at the lid increase the time-averaged Nusselt number in the cavity with respect to the non-pulsating case up to a 16%. This is due to enhancement of the transport phenomena induced by the pulsating flow.

1. Introduction

Low Reynolds number fluid flows are commonly presented in many different engineering applications including, unmanned aerial vehicles, hydrodynamic systems, lubrication, bio-medical systems or convective heat transfer [1]. In this last field, these problems have especial interest in the design of cooling systems for electronic devices where unsteady flow – solid wall interaction is of major importance. The cooling systems that rely on mixed convention across thin fins are a typical example. One of the most promising configurations in this type of mixed convection problems is the use of nanofluids [2]. Nanofluids are basically a colloidal suspension of metal nanosized particles in a fluid. This mixture improves considerably the heat transfer properties of the fluid, enhancing the cooling or heating performances of the system. Nanofluids are specially recommended for cooling systems with limited size such as the ones used in microelectronic devices [3]. Due to this promising potential, heat transfer in nanofluids has been extensively investigated in the last years [4].

There are many contributions dealing with the modeling of the actual heat transfer properties of the nanofluid as a function of the type of nanoparticles and its concentration [5]. However, there are not so many contributions regarding the global characterization of cooling systems with nanofluids in terms of heat transfer correlations. In some cases, such as the mixed-convection thin-fin systems, this flow problem can be well modeled as a heated lid-driven cavity.

The general lid-driven cavity problem has been widely investigated since this configuration can be usually found in many engineering configurations. Some of the first works in the research of the classical lid-driven cavity problem were presented more than twenty years ago for the steady state case [6,7]. From that moment on, the analysis of this configuration has been wide and deep [8]. The fundamental problem of combined forced and natural convection heat transfer in a lid-driven cavity has also received considerable attention from researchers [9–11].

Regarding the nanofluids field, the heated lid-cavity has been studied by different authors for the steady state case [12–16]. However, for the unsteady case, only the contribution from Karimipour et al. [17] is known to the authors. They studied the influence of the Richardson number (Ri) on the heat transfer process when imposing an unsteady, spatially uniform, lid velocity of sinusoidal type and adiabatic vertical walls. They showed that when Ri increases, the time oscillations in the Nu due to the lid velocity oscillation are considerably reduced. Thus, the oscillation effects have relevance in the heat transfer process only for low Ri regimes. If the survey is extended out of the nanofluids field, a few contributions can be found for the same problem. Khanafer et al. [18] studied the heated lid-cavity for a simple fluid of Pr = 0.71 imposing a uniform, time-dependent, velocity lid of sinusoidal type in the range $10^2 < Re < 10^3$, $10^2 < Gr < 10^5$ (i.e. $0.01 < Ri < 0.1$) and

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Nomenclature

ϵ	correction term
ψ	dimensionless streamfunction
ω	dimensionless vorticity
ρ	fluid density
μ	fluid dynamic viscosity
Δ	increment operator
ν	kinematic viscosity of the fluid
ϕ	nanoparticles volume fraction
ψ'	streamfunction
ω'	vorticity
∇^2	Laplacian operator
ρ_{nf}	nanofluid density
μ_{nf}	nanofluid dynamic viscosity
Δx, Δy	length of the volume elements in x and y direction
$\overline{Nu}, \langle Nu_{Top} \rangle$	averaged Nusselt $\overline{Nu} = -\int_0^1 \frac{\partial}{\partial y} dx$
$Re_k = \frac{2f}{\mu k^2}$	vibration Reynolds number of the lid velocity wave
$\tilde{k} = kL$	dimensionless wave number
AR	aspect ratio
C	capacitor
C_{pnf}	nanofluid heat capacitance
C_p	fluid heat capacitance
θ	dimensionless temperature
d_p	particle diameter
f	lid frequency
β	fluid thermal expansion coefficient
τ	fraction of a wave period
g	gravity
G	voltage-controlled current source
J	electric current density
K	fluid conductivity
k	wave number
k_b	Boltzmann constant
K_{nf}	nanofluid conductivity
MEMS	Microelectronic mobile systems
Nu	Nusselt number
PDE	Partial differential equations
Pe = Re·Pr	Pecklet number
R	resistor
Str	$2\pi f \cdot L/u_{ref}$
T	temperature
t,	dimensionless time
t'	time
X, Y	cartesian coordinates
x, y	dimensionless cartesian coordinates
$\mathrm{Gr} = \frac{\beta g \hbar T L^3 \rho^2}{\mu^2}$	Grashof number
$\mathrm{Pr} = \frac{\frac{\mu}{\rho}}{\frac{K}{\rho C_p}}$	Prandtl number
$\mathrm{Re} = \frac{\rho u_{ref} L}{\mu}$	Reynolds number
$\mathrm{Ri} = \frac{Gr}{Re^2} = \frac{\beta g T L}{u_{ref}^2}$	Richardson number with $\Delta T = T_{hot} - T_{cold}$

Subscripts

1, 2, 3	used to identify the voltage-controlled current sources, G
$i \pm \Delta x/2$	right and left ends of the spatial element
i, j	Cartesian index of the center of each element of the spatial discretization
$j \pm \Delta y/2$	upper and bottom ends of the spatial element
ψ	related to dimensionless streamfunction
θ	related to dimensionless temperature
ω	related to dimensionless vorticity
x, y	related to spatial coordinates

$0.1 < (2\pi \cdot \mathrm{Str}) < 5$. They found that, in the studied initial transient lapse, the average Nu decreases with the increase of the lid frequency, whereas the dimensionless drag force increases in amplitude. Chen and Cheng [19] analyzed, for a fluid of Pr = 0.71, the influence of the frequency of the lid velocity in the flow pattern and Nu, in the case of a rectangular heated cavity for Re = 100 and $\mathrm{Gr} = 5 \cdot 10^5$. They identified the natural frequency of the flow and noticed that for lid frequencies over 0.4 times the natural frequency, the lid vibration frequency dominates whereas the natural flow one disappears from the flow pattern spectra. Noor et al. [20], studied the influence of Re and Str on the Nu for a square cavity with double-sided oscillating lids in anti-phase within the range $1 < \mathrm{Re} < 10^3$, Gr = 100 for a fluid of Pr = 0.71. They found that for Re < 10 the lid frequency did not offer significant effect on the flow pattern. Finally, Wan and Kuznetsov [21], analyzed the influence of the aspect ratio (AR) of a non-heated rectangular cavity when a sinusoidal moving wave is imposed as lid at Re = 1000 and Pr = 0.71. They identified this configuration as an excited one and described the influence of the AR on the interactions between primary and secondary eddies of the flow.

All in all, no contribution has been found in the case of pulse or sinusoidal moving velocity waves at the top of a nanofluid, heated, lid-driven cavity. These configurations result to be considerably important, since they are representative of common unsteady pulsating process that might be found in cooling systems of microelectronic devices and MEMS [22,23].

Regarding the modeling of the Navier–Stokes equations in the presence of nanofluids, authors have opted to use different methods for its treatment, such as, finite volumes [14,24], Lattice Boltzman simulations [15,25], Galerkin finite elements[18] or weighting function schemes [13]. A simple method which has been widely applied for modeling transport processes is the NSM [26]. This method obtains a numerical solution through the design and resolution of an electrical circuit network whose mathematical governing equations are formally equivalent to the ones of the problem studied. NSM is specially recommended in the case of non-linear problems [26,27]. Among its applications, it has been shown to behave efficiently not only in the case of heat transfer problems, such as conduction [28], inverse problems [29] or convection in porous media [30], but also in many other transport process problems such as general reaction diffusion systems [31], biological reaction process [32], or even electrochemisty [33].

In this paper, the viability of using the network simulation method to solve viscous, time-dependent, Navier–Stokes equations is analyzed by facing the 2D, incompressible, nanofluid, heated, lid-driven cavity problem. Firstly, a benchmark problem is used to describe and validate this modeling method. Afterwards, new Nusselt correlations are proposed for this nanofluid problem under transient, non-uniform, velocity conditions, i.e. sinusoidal moving velocity waves, at the top of the cavity.

2. Modeling of the Navier–Stokes equations using network simulation method

2.1. Definition of the problem and mathematical formulation

Fig. 1 shows the geometry of the problem analyzed: a 2-dimensional square cavity filled with a Newtonian, incompressible, nanofluid. The base fluid and the nanoparticles are considered to be at thermal equilibrium in an homogeneous mixture. Under these con-