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Time-dependent three-dimensional boundary layer flow of a Maxwell fluid

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ABSTRACT

The present research is concerned with the time-dependent three-dimensional flow of an incompressible Maxwell fluid. The induced flow is due to a stretched sheet. Similarity transformations have been employed for the presentation of the differential systems. The series solutions have been computed by a homotopy analysis method (HAM). Graphical results are shown in order to predict the features of the involved key parameters into the problems.

1. Introduction

The analysis of non-Newtonian fluids has gained much attention because of their obvious industrial and engineering applications. In fact the foams, emulsions, suspensions, polymers, certain oils, etc. cannot be described by the Newtons' law of viscosity and the Navier–Stokes equations are inappropriate for the description of non-Newtonian fluids. Even the boundary layer flows in such fluids under two- and three-dimensional flow situations are very complicated. Thus various recent researchers are engaged in studying different non-Newtonian fluid models under various flow aspects. For-instance Fetecau et al. [1] presented the decay of a potential vortex in a generalized Oldroyd-B fluid. Authors have employed Hankel and Laplace transforms for the development of the solutions which are presented as the sum of Newtonian solutions and the adequate non-Newtonian contribution. Jamil and Fetecau [2] presented some exact solutions for rotating flows of a generalized Burgers' fluid in cylindrical domains. The motion in the fluid is induced due to inner cylinder that applies a time dependent torsional shear to the fluid. Moreover the corresponding solutions for the Burgers', Oldroyd-B, Maxwell, second grade and Newtonian fluids are presented as the special case. Stability analysis of a Maxwell fluid in a porous medium heated from below has been analyzed by Tan and Masuoka [3]. They have analyzed critical Rayleigh number, wave number and frequency for the over stability and found that critical Rayleigh number for over stability decreases as the relaxation time increases whereas porosity acts as an agent to stabilize the system. Hayat et al. [4] presented the magnetohydrodynamic and axisymmetric flow of third grade between stretching sheers in the presence of heat transfer. Authors have utilized similarity transforms for the conversion nonlinear boundary layer equation to the coupled system of ordinary differential equations and employed homotopy analysis method for the computations of the solutions. Mass transfer effects on the unsteady flow of UCM fluid over a stretching sheet has been presented by Hayat et al. [5]. They have modeled the problem for the two-dimensional flow and then employed the homotopy analysis method (HAM) for the construction of the solutions.

The boundary layer flows over a stretching surface are prominent in polymer extrusion, paper production, hot rolling, crystal growing and continuous stretching of plastic films, fiber production, metal extrusion and metal spinning. Sakiadis [6] presented the seminal work on the boundary layer flow over a stretched surface. Since then extensive studies have been performed under various aspects. Literature survey witnesses that majority of the existing information on stretching flow deals with the mathematical analysis in the two-dimensions. However, scarce information is presented for the three-dimensional flow over a stretching

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surface. Ariel [7] found the perturbation and exact solutions for the three-dimensional steady viscous flow past a stretching sheet. He concluded that exact solutions might not possible for the three-dimensional flow of non-Newtonian fluid because of the highly nonlinear relation between stress and deformation rate. Thus researcher should go for the analytic solutions which sufficient amount of accuracy. Mehmood and Ali [8] studied generalized three-dimensional channel flow due to uniform stretching of the plate. Uniform stretching phenomenon has been utilized for the generation of the three-dimensional flow and homotopy analysis method has been utilized for the solution construction. Xu et al. [9] presented the series solutions of unsteady three-dimensional MHD viscous flow and heat transfer in the boundary layer over an impulsively stretching plate. Three-dimensional flow in the Maxwell fluid over a stretching surface has been discussed by Hayat and Awais [10]. They have modeled the three-dimensional momentum equation for the steady flow of Maxwell fluid. A comparison with the previous published results is also shown. Analytic solution for the MHD rotating flow of Jeffery fluid in a channel has been found by Hayat et al. [11]. Authors have concluded that oscillations effects can be generated by incorporating the rotation into the momentum equations. Eldabe et al. [12] presented the three-dimensional flow over a stretching surface in a viscoelastic fluid with mass and heat transfer. As per our knowledge no investigation has been made yet for the time-dependent three-dimensional flow of Maxwell fluid over a bidirectionally stretching surface. In view of this fact the present work has been undertaken. This paper is arranged as follows. Section 2 contains the formulation. Section 3 deals with the series solution by using the homotopy analysis method (HAM) [13–20] and the convergence of the problem is presented in Section 4. Section 5 syntheses the obtained results. Concluding remarks have been reported in Section 6.

2. Formulation of the problem

Consider the time-dependent three-dimensional flow of an incompressible upper convected Maxwell (UCM) fluid bounded by a stretching sheet. The sheet coincides with the plane at $z = 0$ and the flow occupies the region $z > 0$. The motion in fluid is caused by a non-conducting stretching surface as shown in Fig. 1.

The flow is governed by the continuity and momentum equations in the forms

$$\nabla\cdot\mathbf{V} = \mathbf{0}, \tag{1}$$

$$\rho\mathbf{a} = \nabla\cdot\mathbf{T}, \tag{2}$$

in which $\mathbf{V}$ denotes the velocity vector and $\mathbf{T}$ is the Cauchy stress tensor. The acceleration vector $\mathbf{a}$ is defined as

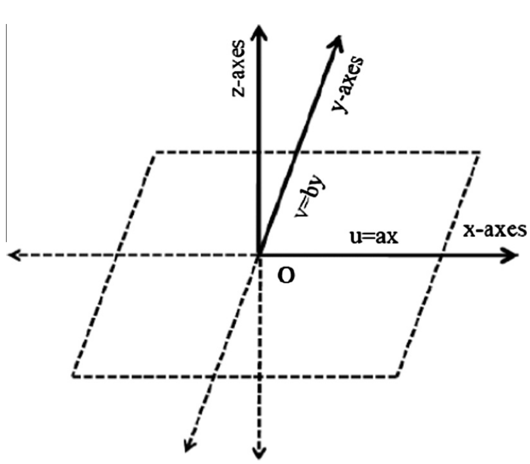

Fig. 1. Geometry of the problem.

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}. \tag{3}$$

The Cauchy stress tensor in Maxwell fluid is given by

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{4}$$

In which an extra stress tensor $\mathbf{S}$ has the relation

$$\left(1+\lambda\frac{D}{Dt}\right)\mathbf{S} = \mu\mathbf{A}_1, \tag{5}$$

where λ indicates the relaxation time, μ is the dynamic viscosity and the first Rivlin–Ericksen tensor $\mathbf{A}_1$ can be expressed as

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \quad \mathbf{L} = \nabla\mathbf{V}, \tag{6}$$

and for a two rank tensor $\mathbf{S}$ we have

$$\frac{D\mathbf{S}}{Dt} = \frac{\partial\mathbf{S}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{S} - \mathbf{L}\mathbf{S} - \mathbf{S}\mathbf{L}^T, \tag{7}$$

From Eqs. (2) and (4) we have

$$\rho\mathbf{a} = -\nabla p + \nabla\cdot\mathbf{S}. \tag{8}$$

Here ρ is the fluid density and p is the pressure. Our interest now is to eliminate $\mathbf{S}$ between Eqs. (5) and (8). Hence applying $(1 + \lambda D/Dt)$ onto Eq. (8), one obtains

$$\rho\left(1+\lambda\frac{D}{Dt}\right)\mathbf{a} = -\left(1+\lambda\frac{D}{Dt}\right)\nabla p + \left(1+\lambda\frac{D}{Dt}\right)(\nabla\cdot\mathbf{S}), \tag{9}$$

Following Harris [21], we use

$$\frac{D}{Dt}(\nabla\cdot) = \nabla\cdot\left(\frac{D}{Dt}\right) \tag{10}$$

and Eq. (9) thus yields

$$\begin{aligned}\rho\left(1+\lambda\frac{D}{Dt}\right)\mathbf{a} &= -\left(1+\lambda\frac{D}{Dt}\right)\nabla p + \nabla\cdot\left(1+\lambda\frac{D}{Dt}\right)\mathbf{S},\\ &= -\left(1+\lambda\frac{D}{Dt}\right)\nabla p + \mu\nabla\cdot\mathbf{A}_1,\end{aligned} \tag{11}$$

For a three-dimensional flow with velocity $\mathbf{V} = [u(x, y, z, t), v(x, y, z, t), w(x, y, z, t)]$, we have in the absence of pressure gradient the following equations in component form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \lambda\begin{pmatrix}\frac{\partial^2 u}{\partial t^2} + 2u\frac{\partial^2 u}{\partial x\partial t} + 2v\frac{\partial^2 u}{\partial y\partial t} + 2w\frac{\partial^2 u}{\partial z\partial t}\\ +u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + w^2\frac{\partial^2 u}{\partial z^2}\\ +2uv\frac{\partial^2 u}{\partial x\partial y} + 2vw\frac{\partial^2 u}{\partial y\partial z} + 2uw\frac{\partial^2 u}{\partial x\partial z}\end{pmatrix}, \tag{12}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \lambda\begin{pmatrix}\frac{\partial^2 v}{\partial t^2} + 2u\frac{\partial^2 v}{\partial x\partial t} + 2v\frac{\partial^2 v}{\partial y\partial t} + 2w\frac{\partial^2 v}{\partial z\partial t}\\ +u^2\frac{\partial^2 v}{\partial x^2} + v^2\frac{\partial^2 v}{\partial y^2} + w^2\frac{\partial^2 v}{\partial z^2}\\ +2uv\frac{\partial^2 v}{\partial x\partial y} + 2vw\frac{\partial^2 v}{\partial y\partial z} + 2uw\frac{\partial^2 v}{\partial x\partial z}\end{pmatrix}, \tag{13}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \lambda\begin{pmatrix}\frac{\partial^2 w}{\partial t^2} + 2u\frac{\partial^2 w}{\partial x\partial t} + 2v\frac{\partial^2 w}{\partial y\partial t} + 2w\frac{\partial^2 w}{\partial z\partial t}\\ +u^2\frac{\partial^2 w}{\partial x^2} + v^2\frac{\partial^2 w}{\partial y^2} + w^2\frac{\partial^2 w}{\partial z^2}\\ +2uv\frac{\partial^2 w}{\partial x\partial y} + 2vw\frac{\partial^2 w}{\partial y\partial z} + 2uw\frac{\partial^2 w}{\partial x\partial z}\end{pmatrix}, \tag{14}$$