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New approaches for computation of low Mach number flows

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ABSTRACT

By using all speed numerical flux schemes, such as SLAU [Simple Low Dissipation AUSM (Advection Upstream Splitting Method)], in MUSCL (Monotone Upwind Scheme for Conservation Laws) approach for compressible CFD, low Mach number flows can be computed without loss of accuracy nor parameter tuning. For an efficient computation, this paper deals with new approaches of implicit time integration method. In this approach, the large sparse matrix system, which consists of flux Jacobian of numerical flux function, has to be solved in each time step. Firstly, a simple Gauss–Seidel iteration method named TC-PGS1(Time Consistent Preconditioned Gauss–Seidel 1) which has flavor of the time derivative preconditioning is introduced. Secondary, we tried to use FGMRES (k) (Flexible Generalized Minimum Residual Method) to solve the non-diagonal dominant linear system arising from Jacobian of flux function SLAU. TC-PGS1 is also used as the matrix preconditioner for FGMRES (k). Optimal parameters for FGMRES (k) is investigated numerically and the performances on computational efficiency of the new methods are compared. It is indicated that FGMRES (k) has apparent advantage on computation of low Mach number flows with sound propagation, however, simpler TC-PGS1 has comparable performance if only flow fields are of interest.

1. Introduction

When compressible CFD methods are applied to low Mach number flows, cares must be taken to excessive numerical dissipation and stiffness due to the large condition number, which is the ratio of maximum and minimum characteristic speeds. Round off error due to very small changes of scalar variables may be another problem, but this can be rather easily avoided by separately storing variables of their reference and variation values as $p = p_\infty + p'$.

The authors proposed all speed numerical flux schemes of AUSM (Advection Upstream Splitting Method) family named SLAU [1] (Simple Low-dissipation AUSM) and showed this scheme can compute from very low to very high Mach number without tuning of parameters, such as the cutoff Mach number. It was also shown that the combination with Weiss–Smith [2] time derivative preconditioning is effective and the stiffness can be avoided at least for steady state flows.

SLAU is a compressible flow CFD algorithm that can compute very low Mach number flows, thus, it can be a good candidate for the direct solver of aero-acoustic problems in the low speed flow, i.e., for solving flow and sound at the same time.

Incompressible flow is thought as low Mach number limit. In incompressible CFD methods pressure wave is neglected, thus acoustics must be treated separately from flow dynamics. If potential methods are used for acoustics, for example, effects of non-uniform velocity field cannot be included easily. As another option, the effect of flow can be reflected by the use of LEE (linearized Euler equation), but still it is difficult when the average flow is hard to be set up. And also the computational cost for LEE is roughly the same as for Euler or laminar Navier–Stokes equation, so the benefit of LEE is not so significant. These are the reasons why we chose the direct solver approach here.

By using explicit time integration, it has been proven in previous research that sound propagation can be computed. However, the usage of explicit schemes is impractical for low Mach number flows since time step determined by sound speed is too restrictive for convection.

A lager time step can be used with implicit time integration. In an implicit method, a large sparse matrix system, which consists of flux Jacobian of numerical flux function, has to be solved in each time step. In our previous work, we introduced Time-Consistent Preconditioned Gauss–Seidel (TC-PGS) [4], a version of preconditioned Gauss–Seidel (GS)[1] implicit time integrations using entropy variables, and demonstrated its accuracy and efficiency over a conventional GS method in solving flow dynamics and aero-acoustics both in low speed flows. The relations between Gauss–Seidel type implicit schemes are illustrated in the upper part of Fig. 1. LU-SGS and MFGS (Matrix Free Gauss–Seidel) in the figure are non precondi-

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[1] The term GS (Gauss–Seidel) is referred to as a GS scheme or a single sweep within a pair of symmetric sweeps in SGS (Symmetric Gauss–Seidel) in this paper.

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Nomenclature

c	sound speed	T	period of wave
e	total energy per unit volume	t	physical time
$\widehat{\mathbf{E}}, \widehat{\mathbf{R}}$	inviscid and viscous flux outward normal to face	u, v, w	Cartesian velocity components
$\widetilde{\mathbf{E}}, \widetilde{\mathbf{R}}$	numerical inviscid and viscous flux outward normal to cell-interface	V_n	velocity component normal to cell-interface
h	total enthalpy	V	volume of cell
L	reference length scale	$\mathbf{W}$	vector of working variables
l	wave length	x_n, y_n, z_n	outward normal of cell-interface
M	Mach number	x, y, z	Cartesian coordinates
$\mathbf{M}$	transforming matrix from conservative to entropy variable	τ	pseudo time
p	pressure	ρ	density
$\mathbf{P}$	preconditioning matrix for linear system	μ	molecular viscosity
$\mathbf{Q}$	vector of conservative variables $(\rho, \rho u, \rho v, \rho w, e)^T$	μ_T	turbulent viscosity
Re	Reynolds number	σ	spectral radius
s	area of cell-interface	*Subscript*	
$\mathbf{S}$	vector of stored field variable	∞	freestream value

tioned schemes suited for higher Mach number flows. The time derivative preconditioning method of Weiss–Smith can be applied to these methods and the preconditioned versions be formulated. TC-PGS is formulated by rearranging the implicit numerical dissipation of the preconditioned schemes. See section 5.2 for the detail.

Through the derivation of TC-PGS, we realized that the essential point of the preconditioned implicit algorithm is to design the implicit numerical dissipation which is matched to the R.H.S with keeping the positive definiteness of the numerical flux Jacobian, which is a necessary and sufficient condition of the diagonal dominance of the linear system. In this study, SLAU is applied as the R.H.S. numerical flux function, thus the apparent choice is to use the implicit dissipation close to that of SLAU. However, the linear system for approximate Jacobian of SLAU turned out to be non-diagonal dominant, as will be explained later in this paper. Traditional iterative linear solvers such as a Gauss–Seidel method are unstable without the diagonal dominance.

Since only an approximate linear solution is required for this purpose, one choice is nevertheless to use diagonal dominant approximation on linear system. This leads to a simpler method named TC-PGS1, which has less dependence on a user-specified parameter, and this will be introduced in this paper first. Then, as another approach, we will solve the non-diagonal dominant system directly by a more sophisticated method, which is shown in the lower part of Fig. 1. In this study, FGMRES (k) by Saad [3] is employed in which we can use different matrix preconditioners at each GMRES step: The SGS matrix borrowed from TC-PGS1 is used as a matrix preconditioner, along with different numbers of iterations at each step.

2. Governing equation and basic numerical scheme

Compressible Navier–Stokes equation is written in integral form as;

$$\iiint \mathbf{Q}_t dv + \oiint (\widehat{\mathbf{E}} - \widehat{\mathbf{R}}) ds = 0 \tag{2.1}$$

By using polyhedrons (polygons in two dimensions) as control volumes, the basic equation for FVM (Finite Volume Method) is written as;

$$\frac{1}{\Delta t}\Delta \mathbf{Q}_i + \frac{1}{V_i}\sum_j (\widetilde{\mathbf{E}}_{i,j} - \widetilde{\mathbf{R}}_{i,j}) s_{i,j} = 0 \tag{2.2}$$

$$\Delta \mathbf{Q}_i = \mathbf{Q}_i^{n+1} - \mathbf{Q}_i^n \tag{2.3}$$

Here subscript i, j means 'j'th face of the cell 'i', n and $n+1$ are physical time steps. Our expression here is based on an unstructured grid formulation, but structured grids can be treated as only a special case.

For unsteady computations, the dual time stepping and 3-point backward Euler scheme are introduced;

$$\frac{\mathbf{Q}_i^{k+1} - \mathbf{Q}_i^k}{\Delta \tau} + \left\{ \frac{\theta_1 \mathbf{Q}_i^{k+1} - \theta_2 \mathbf{Q}_i^n - (\theta_1 - \theta_2)\mathbf{Q}_i^{n-1}}{\Delta t} + \frac{1}{V_i}\sum_j \left(\widetilde{\mathbf{E}}_{i,j}^{k+1} - \widetilde{\mathbf{R}}_{i,j}^{k+1}\right) s_{i,j} \right\} = 0 \tag{2.4}$$

Here, k and $k+1$ denote pseudo time steps. For the second order temporal accuracy with time step variation, coefficients θ_k are given by;

$$(\theta_1, \theta_2) = ((r+2)/(r+1), (r+1)/r) \qquad \Delta t^{n-1} = r\Delta t^n \tag{2.5}$$

where $r = 1$ in this study as usual; for the first order method, they are given by;

$$(\theta_1, \theta_2) = (1, 1) \tag{2.6}$$

The pseudo time τ can be chosen independently from physical time t, and correct time evolution is recovered for any choice of τ when the equation is converged about τ. If non-factored implicit schemes, such as Gauss–Seidel iteration or FGMRES in this paper, are used, faster convergence is obtained by bigger $\Delta\tau$. Thus we took $\Delta\tau$ to be infinitely large, and then, obtained the following equation;

$$\frac{\theta_1 \mathbf{Q}_i^{k+1} - \theta_2 \mathbf{Q}_i^n - (\theta_1 - \theta_2)\mathbf{Q}_i^{n-1}}{\Delta t} + \frac{1}{V_i}\sum_j \left(\widetilde{\mathbf{E}}_{i,j}^{k+1} - \widetilde{\mathbf{R}}_{i,j}^{k+1}\right) s_{i,j} = 0 \tag{2.7}$$

3. Implicit time integration algorithm in delta form

Using first order upwind difference and approximate linearization for L.H.S., implicit time integration scheme of Eq. (2.7) is written as;

$$\left[\frac{\theta_1}{\Delta t} + \frac{1}{V_i}\sum_j s_{i,j}\widetilde{\mathbf{A}}_{i,j}^+\right]\Delta \mathbf{Q}_i - \frac{1}{V_i}\sum_j s_{i,j}\widetilde{\mathbf{A}}_{j,i}^+ \Delta \mathbf{Q}_j = -\mathbf{H}_i^k \tag{3.1}$$

$$\mathbf{H}_i^k = \frac{\theta_1 \mathbf{Q}_i^k - \theta_2 \mathbf{Q}_i^n - (\theta_1 - \theta_2)\mathbf{Q}_i^{n-1}}{\Delta t} + \frac{1}{V_i}\sum_j \left(\widetilde{\mathbf{E}}_{i,j}^k - \widetilde{\mathbf{R}}_{i,j}^k\right) s_{i,j} \tag{3.2}$$