Computers & Fluids 84 (2013) 1–15

Contents lists available at SciVerse ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Numerical simulation of the Graetz problem in ducts with viscoelastic FENE-P fluids

A. Filali, L. Khezzar *

Department of Mechanical Engineering, Petroleum Institute, Abu Dhabi, United Arab Emirates

ARTICLE INFO

Article history:
Received 2 November 2012
Received in revised form 14 March 2013
Accepted 17 May 2013
Available online 29 May 2013

Keywords:
Viscoelastic
Graetz problem
FENE-P model
Heat transfer enhancement
Nusselt number

ABSTRACT

A numerical investigation of heat transfer with viscoelastic fluid based on the FENE-P model in straight pipes of circular and some non-circular cross-sections is carried out to analyze the influence of the rheological parameters on heat transfer enhancement with negligible axial heat conduction and viscous dissipation by assuming temperature-independent model parameters. Numerical simulation is conducted using the finite element based software Polyflow and results are compared with analytical and semi-analytical solutions available in the literature for the 2D axisymmetric pipes. The analysis considers a constant wall heat flux boundary condition and shows that an increase in fluid elasticity raised the normalized heat transfer coefficient due to the increased level of shear-thinning behavior. But increasing the extensibility parameter L^2 leads to a decrease in Nusselt number. Nusselt number distribution in the entrance region of tubes of equilateral triangular, square and rectangular cross-sections is reported for the first time for non-linear viscoelastic fluids of the FENE-P type.

1. Introduction

Laminar, thermal entry flow problems in ducts are relevant to a number of important practical applications. Such a problem is referred to in the classical literature as the Graetz problem. While for Newtonian fluids and laminar flow, solutions have been well documented in Shah and London [1] and Shah and Bhatti [2] for a variety of different duct cross sections, the same is not true for non-Newtonian visco-elastic fluids. Nonetheless, heat transfer enhancement in laminar flow in tubes of non-circular cross sections has been recognized previously by Hartnett and Kostic [3].

Theoretical solutions for flow and for thermal entry heat transfer in axisymmetric pipes and two-dimensional plane duct have been obtained using the simplified version of the Phan-Thien and Tanner (PTT) constitutive equation see Oliveira and Pinho [4], Coelho et al. [5,6]. In [5], the SPTT model was investigated for the Graetz problem for tube and plane cases for constant wall temperature thermal boundary condition. Two cases of the Graetz problem have been investigated with and without viscous dissipation. An exact solution was developed for the first one and a semi-analytic solution for the second one. In [6], a theoretical solution for flow in a tube and in a plane duct for both constant wall temperature thermal boundary condition and constant wall heat flux in the presence of viscous dissipation. The solution was obtained using the hydrodynamic solution developed by Oliveira and Pinho [4] and the method of separation of variables and the results are presented in terms of the effect of We and ε and the viscous dissipation on the Nu variation. Recently, a numerical simulation of the Graetz problem for SPTT fluid trough ducts of circular, square, rectangular and triangular cross sections under constant heat flux and wall temperature thermal boundary conditions has been conducted by Filali et al. [7] using the finite element ANSYS Polyflow code. The results are presented and discussed in terms of the combined effects of elasticity (through the Weissenberg number) defined as $We \equiv \lambda\bar{u}/R$, where λ is the relaxation time and $\bar{u}$ is the average velocity, and elongational parameter (through the PTT parameter ε) on the velocity profile and the Nusselt number. The results have highlighted the difficulty to simulate flows with high elasticity We. Spectral and pseudo-spectral based techniques of integration may offer an alternative without such limitations at higher values of We [8,9]. These techniques which can produce stable discretization of both elliptic and hyperbolic linear problems [10], have found extensive use in a wide variety of fluid mechanics problems requiring high accuracy approximations, see [11,12] and for steady-state viscoelastic flows [13]. But it has also to be recognized that for non-linear viscoelastic problems, spectral approximations can introduce instabilities in the discrete set of equations [12], though these can be removed by using filtering techniques [14]. Alternative approaches, which combine the high accuracy of the spectral methods with the computational efficiency of finite element/finite difference techniques by using a mixed spectral/finite element or finite difference interpolation [15–17] have shown considerable success

* Corresponding author. Tel.: +971 50 820 1536.
E-mail address: lkhezzar@pi.ac.ae (L. Khezzar).

0045-7930/$ - see front matter
http://dx.doi.org/10.1016/j.compfluid.2013.05.013

Nomenclature

ρ	fluid density (kg/m^3)
c_p	specific heat (j/kg K)
k	thermal conductivity (w/mK)
Re	Reynolds number ($\rho\bar{u}D/\mu$)
Nu	Nusselt number (hD/k)
T	temperature (K)
q	heat flux (w/m^2)
r	radial coordinate (m)
R	tube radius (m)
D	pipe diameter (m)
x	axial coordinate (m)
x'	normalized axial coordinate ($x/DRePr$)
y, z	transversal coordinates (m)
u	velocity (m/s)
$\bar{u}$	average velocity (m/s)
Q	flow rate (m^3/s)
We	Weissemberg number ($\lambda\bar{u}/R$)
Pr	Prandtl number ($\mu c_p/k$)
L^2	extensibility parameter
C_f	skin friction coefficient
C_{fN}	skin friction coefficient of Newtonian fluid

Greek symbols

μ	fluid viscosity (Pa s)
θ	non-dimensional temperature
λ	relaxation time (s)
τ	stress tensor

in viscoelastic fluid flow simulations albeit at a high computational cost due to the required size of the meshes employed in order to achieve convergence.

Another common viscoelastic model for polymeric fluids is the FENE-P model (Finite Extensible Nonlinear Elastic while *P* stands for the closure proposed by Peterlin, see [18,19]). The FENE-P model is one of the most commonly used models for polymeric liquids in the form of dilute, semi-dilute and concentrate solutions, see Purnode and Crochet [20]. In contrast to the PTT and SPTT models which are continuum based models, the FENE-P possesses a molecular based constitutive equation and is known to predict decreasing viscosity with shear and accurately model viscometric properties for a number of fluids.

A fully developed solution for pipe and plane two-dimensional flow of a FENE-P fluid has been derived by Oliveira [21]. It was found that the velocity profile becomes flatter as the dimensionless parameter characterizing the viscoelasticity, the Deborah number (De), increased and the extensibility parameter L^2 for FENE-P model decreases. This is due to the shear-thinning behavior of the FENE-P fluid. The normal stress profile was found to vary in a non-monotone way with De. For $De < De_{cr}$ (depend on L^2), the normal stress increases with elasticity, but above De_{cr} this trend is reversed.

Assuming constant thermal properties and using the fully developed isothermal solution obtained by Oliveira [21], Oliveira et al. [22] obtained a semi-analytical solution for the Graetz problem of highly viscoelastic liquids of the FENE-P constitutive type in pipes and channels including viscous dissipation under prescribed constant wall temperature and wall heat flux thermal boundary conditions. They determined that the difference between the behavior of the PTT and FENE-P fluids in this shear flow lies in the effects of ε and the extensibility parameter L^2, respectively. Increases in both ε and We lead to an increase of the Nusselt number as in the work of Coelho et al. [5]. L^2 and $\varepsilon^{1/2}We$ have opposite effects on heat transfer enhancement. Increases in the parameter $\varepsilon^{1/2}We$ lead to enhanced Nusselt numbers and higher values of L^2 lead to decreased shear-thinning and heat transfer.

Thus, analytical solutions have been developed for the Graetz problem for viscoelastic fluids using simple geometric configurations as the axisymmetric pipe and plane two-dimensional duct and cannot be used for complex three-dimensional ducts. Nonetheless they serve to validate solutions obtained by numerical models.

The objective of the present work is to extend the work of Oliveira and Oliveira et al. [21,22] and investigate numerically using the finite element based software Polyflow [23], the Graetz problem in tubes of selected non-circular cross-section using the non-linear viscoelastic FENE-P constitutive equation. The simulations consider first the 2D axisymmetric geometry. This geometry is chosen because an analytical solution exists for it [21,22]; it will hence serve to validate the numerical procedure. The study will then be extended to other non-axisymmetric geometries such as ducts with triangular, square and rectangular cross sections. The study will analyze the influence of the rheological parameters on the heat transfer enhancement with negligible axial heat conduction and viscous dissipation. The analysis considers the influence of the dimensionless parameter characterizing the viscoelasticity, the Weissenberg number (We), and the extensibility parameter of the model L^2 on the thermal and flow field behavior. Numerical results for the non-circular geometries are presented here for the first time.

2. Mathematical model and Nusselt number

The field conservation equations for mass, momentum and energy for steady incompressible laminar flow and the various constitutive equations are given below.

Mass conservation:

$$\vec{\nabla}\cdot\vec{u} = 0, \tag{1}$$

Momentum conservation:

$$\vec{\nabla}\cdot(\rho\vec{u}\cdot\vec{u}) = -\vec{\nabla}p + \vec{\nabla}\cdot\tau, \tag{2}$$

Energy equation:

$$\vec{\nabla}\cdot(\rho c_p\vec{u}T) = \vec{\nabla}\cdot(k\vec{\nabla}T), \tag{3}$$

where $\vec{u}$ represents the velocity vector, p the pressure, T the temperature and τ the total extra-stress tensor. The properties: ρ the fluid density, k the thermal conductivity and c_p the specific heat are assumed constant in line with the studies reported [21,22]. In real experiments however, the thermal properties cannot be considered independent of temperature and can have an effect, see Nóbrega et al. [24]. In the present study however, transport and thermodynamic properties, such as viscosity, thermal conductivity, density and specific heat, are considered independent of temperature. Variation of fluid properties with temperature can account for important differences, but leads to a more complex problem requiring a detailed study which is left for future investigations.

For viscoelastic flows, the total extra-stress tensor is decomposed into a viscoelastic component $\boldsymbol{\tau}_1$ and a purely-viscous component $\boldsymbol{\tau}_2$:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2, \tag{4}$$