Computers & Fluids 42 (2011) 82–91

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Numerical simulations of particle migration in a viscoelastic fluid subjected to Poiseuille flow

M.M. Villone [a], G. D'Avino [b], M.A. Hulsen [c], F. Greco [d], P.L. Maffettone [a,*]

[a] Dipartimento di Ingegneria Chimica, Universitá di Napoli Federico II, Piazzale Tecchio 80, 80125 Napoli, Italy
[b] Department of Chemical Engineering, K.U. Leuven, W. de Croylaan 46, B-3001 Leuven, Belgium
[c] Department of Mechanical Engineering, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands
[d] Istituto di Ricerche sulla Combustione, IRC-CNR, Piazzale Tecchio 80, 80125 Napoli, Italy

ARTICLE INFO

Article history:
Received 22 April 2010
Received in revised form 6 September 2010
Accepted 5 November 2010
Available online 16 November 2010

Keywords:
Particle migration
Microfluidic
Viscoelasticity
Poiseuille flow
Numerical simulations
Arbitrary Lagrangian Eulerian formulation

ABSTRACT

In this work we present 2D numerical simulations on the migration of a particle suspended in a viscoelastic fluid under Poiseuille flow. A Giesekus model is chosen as constitutive equation of the suspending liquid. In order to study the sole effect of the fluid viscoelasticity, both fluid and particle inertia are neglected.

The governing equations are solved through the finite element method with proper stabilization techniques to get convergent solutions at relatively large flow rates. An Arbitrary Lagrangian–Eulerian (ALE) formulation is adopted to manage the particle motion. The mesh grid is moved along the flow so as to limit particle motion only in the gradient direction to substantially reduce mesh distortion and remeshing.

Viscoelasticity of the suspending fluid induces particle cross-streamline migration. Both large Deborah number and shear thinning speed up the migration velocity. When the particle is small compared to the gap (small confinement), the particle migrates towards the channel centerline or the wall depending on its initial position. Above a critical confinement (large particles), the channel centerline is no longer attracting, and the particle is predicted to migrate towards the closest wall when its initial position is not on the channel centerline. As the particle approaches the wall, the translational velocity in the flow direction is found to become equal to the linear velocity corresponding to the rolling motion over the wall without slip.

1. Introduction

In many practical processes, particles are suspended in fluids in order to give specific properties to the final composite material (for example filled polymers, paints, coatings). On the other hand, in several systems, particles are naturally transported in fluids (i.e. cells in blood, pollutants in gaseous flows, etc.). Quite often, the channel dimensions where suspensions flow are comparable to the particles size (i.e. microfluidic devices), and the suspending liquid exhibits viscoelastic effects such as normal stresses and shear-rate dependence of the viscosity.

It is well known that particles suspended in liquids can show peculiar effects with a phenomenology strongly depending on the type of the flow as well as on the fluid rheology. One such effect, of interest here, is cross-streamline migration, i.e., the motion of the particle orthogonally to the direction of the main flow. This problem received great interest over the last 50 years.

The first experimental observations on the migration phenomenon were performed by Segré and Silberberg [1,2]. The authors studied the macroscopic inertial migration of non-interacting, neutrally buoyant spheres in a Newtonian fluid in a tube flow. They found that the particles migrate away from both the wall and the channel centerline, and move towards an *equilibrium* radial position of about 0.6 times the tube radius.

Later on, the relevant phenomenology observed by Segré and Silberberg [1,2] has been experimentally confirmed in several works [3–5]. Recently, the cross-streamline migration has been studied at moderately high Reynolds numbers (up to 2500) by Matas et al. [6]. They observed a motion towards the wall of the equilibrium radial position found by Segré and Silberberg as the Reynolds number is increased. The same observations were made by Eloot et al. [7] where convective transport of neutrally buoyant spherical particles is studied in capillaries using an on-line particles detector.

* Corresponding author. Tel.: +39 0817682282; fax: +39 0812391800.
E-mail addresses: massimiliano_villone@hotmail.com (M.M. Villone), gadavino@unina.it (G. D'Avino), m.a.hulsen@tue.nl (M.A. Hulsen), fgreco@irc.na.cnr.it (F. Greco), pierluca.maffettone@unina.it (P.L. Maffettone).

0045-7930/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compfluid.2010.11.009

Perturbation analysis has been widely used to study the effect of the inertia on the particle migration in shear and Poiseuille flows giving satisfactory agreement with the experimental observations [8–10]. Direct numerical simulations have also been performed in order to examine the effect of a finite Reynolds number on the cross-streamline migration. An extensive literature has been produced, focusing on several aspects such as neutrally and non-neutrally buoyant particles [11,12], lift-off correlations [13], existence of multiple equilibrium positions [14]. Recently, the analysis has been extended to 3D systems confirming the phenomenology experimentally observed [15–17]. For a comprehensive review of the inertial effects on the particle migration the reader can refer to [17] and the references therein.

Concerning the effect of the viscoelasticity of the suspending fluid, migration of solid spherical particles at low Reynolds number in non-Newtonian fluids has been studied experimentally by Mason and co-workers [18–20]. The results showed that the magnitude and direction of migration is strictly dependent on the rheological properties of the suspending medium. For an essentially non-elastic, shear-thinning fluid they observed that neutrally buoyant solid spheres migrate toward the wall in Poiseuille flow. On the contrary, in an viscoelastic medium the migration in the minimum shear-rate direction in Poiseuille flow (i.e. toward the centerline) was observed. However, it should be mentioned that the rheological data were incomplete in these works.

Recently, the cross-streamline migration of bubbles in a viscoelastic channel flow has been studied [21]. When a surfactant is used, making the particle–fluid interface rigid, a transverse migration is observed with direction towards the channel walls. It must be remarked that in these experiments the particle size is comparable with the channel dimension.

By using a perturbative method, Ho and Leal [22] developed an analytical theory by considering a Second Order Fluid as suspending medium. They found that, due to the normal stresses, a particle migrates in the direction of decreasing absolute shear rate, which is towards the axis channel in Poiseuille flow.

Joseph and co-workers [12–14] made 2D direct numerical simulations taking into account the viscoelasticity of the suspending liquid, modeled as an Oldroyd-B fluid with a Bird–Carreau shear-rate viscosity dependence. They found that the migration direction depends on the competition of inertia, blockage ratio, elasticity and shear thinning of the fluid. Limiting to the Poiseuille flow case, they indicate that the elasticity of the fluid drives the particle towards the axis of the channel, whereas shear thinning makes the particle migrate towards the closest wall.

In this work we perform a systematic numerical study on the migration of a particle suspended in a viscoelastic fluid under Poiseuille flow at moderately large flow rates (as compared with [12]). The suspending medium is modeled as a Giesekus fluid [23], a model often capable of accurately describing experimental viscoelastic data. The study is carried out by neglecting fluid and particle inertia, and the analysis is performed through 2D Direct Numerical Simulations. The influence of the flow rate, particle dimension (as compared to the gap size) and the shear thinning on the migration velocity are investigated.

The momentum balance is discretized through the DEVSS (Discrete-Elastic-Viscous-Split-Stress) method that is one of the most robust formulations currently available. The viscoelastic constitutive equation is stabilized by implementing the SUPG (Streamline-Upwind-Petrov-Galerkin) technique. Furthermore, a log-conformation representation of the conformation tensor is used. Finally, an ALE particle mover [24] is adopted to handle the particle motion. To easily manage the particle motion, the mesh grid is translated along the flow direction with a velocity equal to the particle x-velocity. Consequently, the relative x-distance between the mesh nodes and the particle is kept unchanged and the particle only moves along the y-direction (i.e., the migration direction). In this way, remeshing due to ALE approach is only needed once-twice per simulation, always preserving the accuracy of the solution.

2. Governing equations

In Fig. 1a a schematic diagram of the problem is presented: a single, rigid, non-Brownian, inertialess, circular particle (2D problem) moves in a channel filled by a viscoelastic fluid in Poiseuille flow. The particle with diameter $D_p = 2R_p$, denoted by $P(t)$ and boundary $\partial P(t)$, moves in a rectangular domain, Ω, with dimensions L and H along x- and y-axis respectively and external boundaries denoted by Γ_i ($i = 1,\ldots,4$). The Cartesian x and y coordinates are selected with the origin at the center of the domain. On the upper and lower boundaries, no-slip conditions are set whereas a flow rate Q is imposed on the left (inflow) boundary. Finally, periodicity is imposed on the left and right boundaries. For an unfilled Newtonian fluid, this would generate the well-known parabolic velocity profile depicted in Fig. 1b (dashed line). In the figure, the maximum velocity $u_{\max}$ and the average velocity $\bar{u}$ are also reported. The solid line is the velocity profile for a viscoelastic fluid with a constitutive equation and model parameters chosen as discussed below and with the same flow rate as the Newtonian fluid.

The vector $\boldsymbol{x}_p = (x_p, y_p)$ gives the position of the center of the particle P whereas the particle angular rotation is denoted by $\boldsymbol{\Theta} = \Theta\boldsymbol{k}$, where $\boldsymbol{k}$ is the unit vector in the direction normal to the x–y plane. The particle moves according to the imposed flow and its rigid-body motion is completely defined by the translational velocity, denoted by $\boldsymbol{U}_p = d\boldsymbol{x}_p/dt = (U_p, V_p)$ and angular velocity, $\boldsymbol{\omega} = d\boldsymbol{\Theta}/dt = \omega\boldsymbol{k}$.

The governing equations for the fluid domain, $\Omega - P(t)$, neglecting inertia, read as follows:

$$\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{0} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta_s\boldsymbol{D} + \boldsymbol{\tau} \tag{3}$$

Eqs. (1)–(3) are the momentum balance, the mass balance (continuity) and the expression for the total stress, respectively. In these equations $\boldsymbol{\sigma}$, $\boldsymbol{u}$, p, $\boldsymbol{I}$, η_s, $\boldsymbol{D}$, are the stress tensor, the velocity vector, the pressure, the 2×2 unity tensor, the viscosity of a Newtonian 'solvent', and the rate-of-deformation tensor, respectively. The viscoelastic stress tensor, $\boldsymbol{\tau}$, is written as (for the constitutive model chosen, see below):

$$\boldsymbol{\tau} = \frac{\eta}{\lambda}(\boldsymbol{c} - \boldsymbol{I}) \tag{4}$$

where $\boldsymbol{c}$ is the 'conformation tensor', η is the polymer viscosity, and λ is the relaxation time.

We will model the viscoelastic fluid with the Giesekus constitutive equation (for $\boldsymbol{c}$):

$$\lambda \overset{\nabla}{\boldsymbol{c}} + \boldsymbol{c} - \boldsymbol{I} + \alpha(\boldsymbol{c} - \boldsymbol{I})^2 = \boldsymbol{0} \tag{5}$$

where α is the so-called mobility parameter that modulates the shear thinning behavior. The symbol ($^{\nabla}$) denotes the upper-convected time derivative, defined as:

$$\overset{\nabla}{\boldsymbol{c}} \equiv \frac{\partial \boldsymbol{c}}{\partial t} + \boldsymbol{u} \cdot \nabla\boldsymbol{c} - (\nabla\boldsymbol{u})^T \cdot \boldsymbol{c} - \boldsymbol{c} \cdot \nabla\boldsymbol{u} \tag{6}$$

Notice that the zero-shear-rate viscosity is given by $\eta_0 = \eta_s + \eta$.

The boundary and initial conditions are:

$$\boldsymbol{u} = \boldsymbol{U}_p + \boldsymbol{\omega} \times (\boldsymbol{x} - \boldsymbol{x}_p) \quad \text{on } \partial P(t) \tag{7}$$

$$u = v = 0 \quad \text{on } \Gamma_1 \text{ and } \Gamma_3 \tag{8}$$

$$\boldsymbol{c}|_{t=0} = \boldsymbol{c}_0 \tag{9}$$