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Coupling superposed 1D and 2D shallow-water models: Source terms in finite volume schemes

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A B S T R A C T

We study the superposition of 1D and 2D shallow-water equations with non-flat topographies, in the context of river-flood modeling. Since we superpose both models in the bi-dimensional areas, we focus on the definition of the coupling term required in the 1D equations. Using explicit finite volume schemes, we propose a definition of the discrete coupling term leading to schemes globally well-balanced (the global scheme preserves water at rest whatever if overflowing or not). For both equations (1D and 2D), we can consider independent finite volume schemes based on well-balanced Roe, HLL, Rusanov or other scheme, then the resulting global scheme remains well-balanced. We perform a few numerical tests showing on the one hand the well-balanced property of the resulting global numerical model, on the other hand the accuracy and robustness of our superposition approach. Therefore, the definition of the coupling term we present allows to superpose a local 2D model over a 1D main channel model, with non-flat topographies and mix incoming-outgoing lateral fluxes, using independent grids and finite volume solvers.

1. Introduction

In river hydraulics, operational models are generally based on the St-Venant equations (1D shallow-water). If overflowing, flood plain are represented in the 1D model by storage areas, that are defined by using empirical laws and/or terms to be calibrated, see [6] or e.g. [14]. Obviously, flow dynamic inside the storage areas is not computed; also the empirical laws can be difficult to calibrate. If for any reason, the end-user has to model the flow in the flood plain, a 2D model must be used, see e.g. Fig. A.1. Then, the classical approach is to decompose the domain, re-define the mesh, then couple the 1D model (in the non-flooded areas) with 2D models (in flooded areas) at interfaces, see e.g. [15,13]. Coupling conditions have to be imposed at interfaces only. An efficient coupling procedure may be a Schwarz-like algorithm. Nevertheless, this approach presents some drawbacks. It requires to re-define the 1D hydraulic model (mesh, boundaries, etc.) and very probably the related topography data. The 1D model (which is potentially a complex network) must be segmented (decomposed) in order to combine it with the 2D models. It can be a heavy task.

A superposition approach is proposed in [8,12]. In such an approach, instead of decomposing the original 1D (network) model, one superposes the 2D model (so-called "local zoom model"). The superposition approach presents some advantages. The original 1D model remains intact and the 2D local models can be performed with their own dynamics (typically, time steps and mesh grids are much smaller for 2D solvers than for 1D solvers). Nevertheless, an accurate definition of the coupling terms between both models is required. At interfaces, incoming characteristics are still good conditions, but along the 1D main channel one must introduce a coupling term in the 1D equations (modelling the loss or gain of mass and momentum). This coupling term has to take into account the outgoing and incoming fluxes if overflowing. From a continuous point of view, the coupling source term can be derived formally from the 3D Navier–Stokes equations, see [12].

Then, next step is to define a stable and well-balanced global scheme. An important difficulty is to discriminate between the 1D-topography graph $Z_b(\tilde{x})$ and the 2D-topography graph $z_b(x,y)$, since Z_b depends on the curvilinear coordinate $\tilde{x}$ while z_b depends on the cartesian coordinates (x,y). In addition for real cases, data are sparse, uncertain, and the 1D-topography and the 2D-topography do not have to respect the same hydrological constraints. If in addition, one wants to consider different meshes and schemes for the 1D model and the 2D model, the discretization of the coupling source term must be such that it leads to a consistent, stable and well-balanced scheme. This is the problem we address in the present study, while focusing on explicit finite volume schemes.

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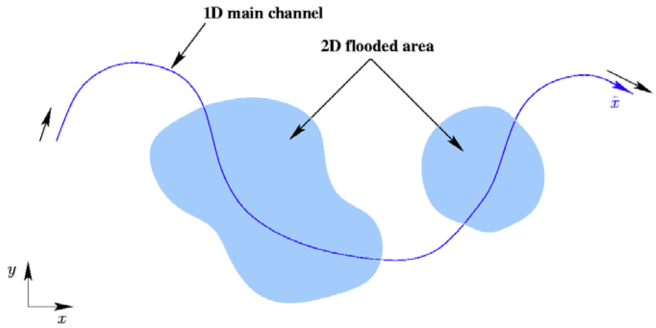

Fig. A.1. Modeling outline: a global 1D model with superposed local 2D models.

Let us point out that we present the 1D–2D coupling in the context of river hydraulics, but this could also be apply to any other flows involving 1D and 2D shallow-water equations with non-flat topographies.

We consider the possibility of using different finite volume schemes for the 1D model and the 2D model. In the numerical analysis presented in next sections, they can be based on different time-space grids but they must be explicit in time. More precisely, we consider finite volume methods in conservative form with source terms (the topography terms and the coupling term). Then, for both models, we can consider any solver belonging to a whole family of approximate Riemann solvers. We prove that the resulting global scheme is well-balanced in the sense that it preserves water at rest, with and without overflowing.

We present some numerical results for an academic test case with a non-constant topography in which there are outgoing and incoming lateral fluxes. In order to couple the models, we use a Schwarz coupling algorithm (global in time). This could be done also by using an optimal control approach as in [8,12]. The numerical results show that after convergence, the coupling source term Ψ defined in the present study leads to a global solution as accurate as a full 2D solution in case of matching grids, and leads to a robust and accurate solution if grids are mismatching.

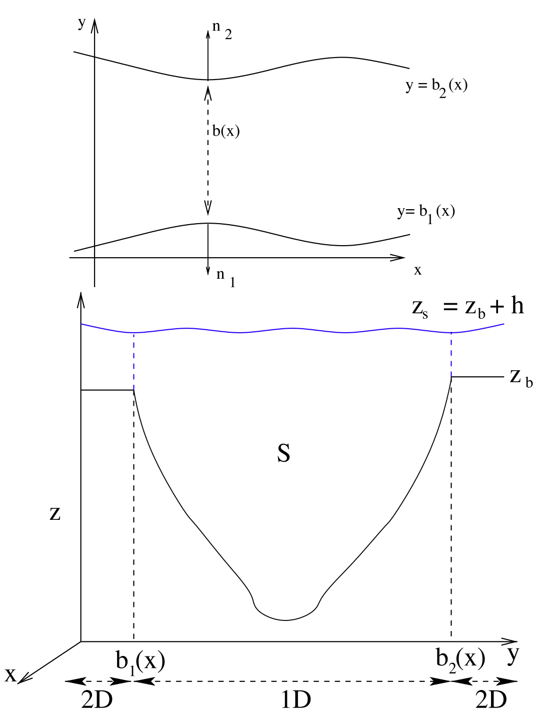

Fig. A.2. Up: definition of the 1D channel in the 2D domain. Down: 1D cross-section with overflowing.

The paper is organized as follows. In Section 2, we present the two mathematical models. Their discretization using well-balanced finite volume scheme is presented in Section 3. The discretization of the coupling source term in 1D equations is described in Section 4. We begin with the simplest case (matching grids and 1D linear axis). Then, we consider the case of 1D curvilinear geometry with matching grids. Finally, the most general case (curvilinear and mismatching grids) is considered. We prove in Theorem 1 that the here introduced discrete source term leads to a global well-balanced scheme, whatever the choice of the well-balanced finite volume method used for the 1D and 2D models. In Section 5 we present some numerical experiments to validate the definition of the discrete source term, and to show the efficiency of the present superposition approach. We recall briefly in Appendix A the derivation of the coupling source term in the 1D equations from the 3D Navier–Stokes equations (we refer to [12] for more details).

2. Mathematical models

2.1. The 1D model with source term

The 1D model is based on St-Venant equations (1D shallow-water equations). Nevertheless, since our goal is to couple this 1D model to a 2D shallow-water model, we must take into account transfers through the two lateral boundaries of the main channel. If we integrate the 3D Navier–Stokes equations over the vertical wetted area, in the presence of lateral transfer terms, we obtain some source terms in the 1D equations. The derivation of these source terms is presented in Appendix A. The result is the following. Let us denote the channel-following coordinates by: $\tilde{x}$. We denote the unidimensional variables (i.e. depending on $(\tilde{x}, t)$ only) as follows: S the wet cross-section, Q the discharge, H the water depth. And $\mathscr{Z}_b$ denotes the unidimensional topography (depending on $\tilde{x}$ only). We assume that: *the channel width variations are small, u is nearly constant over the cross-section, and (u, v) does not depend on z on boundaries b_1 and b_2*. Furthermore, for the sake of simplicity, we consider *rectangular cross-sections* only. Hence $S = b \cdot H$, where b is the channel width, see Fig. A.3.

The derivation of the 1D shallow-water equations with source term is presented in Appendix A (replace x by $\tilde{x}$). Under the assumptions above, the equations are the following:

$$\begin{cases} \frac{\partial S}{\partial t} + \frac{\partial Q}{\partial \tilde{x}} = -(q_{\eta_1} + q_{\eta_2}) \\ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial \tilde{x}}\left(\frac{Q^2}{S} + P\right) - g\frac{\partial b}{\partial \tilde{x}}\frac{H^2}{2} + gS\frac{\partial \mathscr{Z}_b}{\partial \tilde{x}} = -(q_{\eta_1}u_{t_1} + q_{\eta_2}u_{t_2}) \end{cases} \tag{1}$$

where $P = gS\frac{H}{2}$ is a pressure term. In the right hand side, q_{η_i} represents the discharge normal to the lateral boundary i of the main channel, $i = 1, 2$; u_{t_i} represents the tangential velocity at lateral

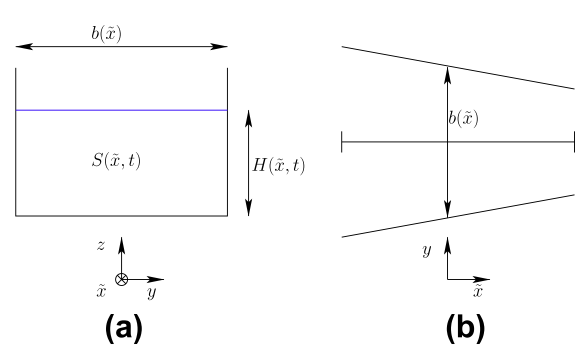

Fig. A.3. 1D model. (a) Wet cross-section; (b) top view of one 1D cell in the main channel.