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Boundary treatment for the lattice Boltzmann method using adaptive relaxation times

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ABSTRACT

In this paper, we propose a new boundary treatment with almost second-order accuracy that does not require neighboring lattice information. In order to achieve improved accuracy for the boundary lattices, we used adaptive relaxation times reflecting boundary length scales that were unequal to the length scale of the internal fluid region lattices. Since the boundary treatment using adaptive relaxation times at the boundaries was formulated without information about the neighboring lattices, it could be easily applied to complex geometries. Numerical results using the proposed boundary treatment showed almost second-order accuracy for two-dimensional and three-dimensional problems without using information from neighboring lattices, unlike interpolation or extrapolation methods.

1. Introduction

The lattice Boltzmann method (LBM) is a computational method based on the dynamics of particles used for solving engineering problems governed by partial differential equations. The LBM has already achieved great success in computationally simulating fluid flow, and has applied in diverse problems with complex geometries [18,28]. Beyond the fluid dynamics solver, Qian et al. [11] derived the general reaction–diffusion equations for the LBM. Jawerth et al. [21] presented two lattice Boltzmann models for nonlinear anisotropic diffusion of images. Zhao [40] used the LBM to solve the Laplace and Poisson equations for the diffusion process in image processing. Yan [25] proposed the LBM for the wave equation. Chopard and Luthi modified the LBM to simulate wave propagation in complex environments and fracture processes [20]. Wang et al. [36–39,45] used the LBM to solve the energy transport equation with complex multiphase porous geometries, and established a method to predict material properties such as the effective thermal conductivities of porous media. Li and Hi [43] combined the LBM with the finite difference method to simulate incompressible, resistive magnetohydrodynamic flows. Guo et al. [46] and Verhaeghe et al. [47] improved LBM to apply to microflows considering slip boundary and large Knudsen number.

As applications for the LBM expanded, the LBM has been developed to increase the accuracy and stability. The multiple-relaxation-time (MRT) LBM was developed by many researchers [7,23,26,29,30] to overcome drawbacks of the simple LBM that is known as the lattice Bhatnagar–Gross–Krook (LBGK) scheme [1,3,5]. Lallemand and Luo [23] showed an improvement in numerical stability with the use of the MRT LBM. d'Humières et al. [29] confirmed the stability improvement in three-dimensional lid-driven cavity flow using the MRT LBM. Diverse boundary treatments were also developed. Traditionally, the LBM used a bounce-back treatment for the wall boundary condition. The bounce-back boundary treatment requires that the boundaries should be located midway through the last fluid node and the first outside node for second-order accuracy [8,9]. Otherwise, the bounce-back boundary treatment gives only first-order accuracy at the boundaries. This constraint made the bounce-back boundary treatment insufficient for complex boundaries. To overcome the limits of this boundary treatment, many modified boundary treatments have been applied to complex boundaries. Some used an extrapolation and/or interpolation (link-type) boundary treatment [13,22,24,27,31,33,34]. Guo and Zheng [31] used the extrapolation of non-equilibrium distributions on a curved boundary and achieved good second-order accuracy. Bouzidi et al. [27] established a well-organized interpolation for the unknown distribution of a curved boundary and also achieved second-order accuracy. Lallemand and Luo [33] expanded on the work of Bouzidi et al. to moving boundaries. Ginzburg et al. [32,41,42] suggested general theoretical tools for studying the existing link-type boundary treatments and proposed multireflection boundary treatment with third order kinetic accuracy. To enhance the stability of the mass-conservation equation, they also used two eigenvalues with eigenvectors for the collision matrix. These kinds of boundary treatments are simple and easy to use in most cases. However, these interpolation and/or extrapolation boundary treatments require additional

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information from the neighboring lattices. Especially in corner boundaries, there is a lack of neighboring lattices, so the boundary treatments may need some assumptions and/or precise treatments according to each situation [41]. Additionally, these interpolation and/or extrapolation boundary treatments can show irregular velocity profile by the difference of the dissipation due to the interpolation or extrapolation around the boundary [24]. Other studies have proposed modified bounce-back boundary treatments, such as the half-away bounce-back method [6] and the non-equilibrium bounce-back method [16]. The half-away bounce-back boundary treatment does not accurately reflect complex geometries because it uses stair-step boundaries. In addition, a hydrodynamic approach [10] was proposed, but applied only to confined cases. Filippova and Hanel [19] proposed a grid refinement scheme around boundaries to enhance the local resolution for better accuracy and higher Reynolds number. They adopted a hierarchical refinement concept by introducing different relaxation times between the coarse and fine grids using Chapman–Enskog expansion and Taylor series of Knudsen number. Rohde et al. [44] proposed volumetric techniques for grid refinement and boundary conditions to be applied to the flow around a sedimenting sphere in a tank. Their numerical results showed good agreement with experimental results. These grid refinement schemes locally added lattice points near the boundaries to enhance accuracy and resolution. However, the boundary condition for lattice points adjacent to boundaries were applied by using the conventional boundary treatments and have similar problems. Guo et al. [46] investigated the physical symmetry, spatial accuracy, and relaxation time for microgas flows with LBM. They issued the wall effect on the relaxation time. Guo et al. [46] and Verhaeghe et al. [47] showed that diverse approaches on relaxation time are required to solve accurately the Dirichlet boundary conditions.

In this paper, we propose a new boundary treatment with almost second-order accuracy at the boundaries even without using information from neighboring lattices. The proposed treatment do not add more lattice points near the boundary. The proposed boundary treatment modifies only the collision scheme with adaptive relaxation times that reflect the different length scales in the boundary lattices. Our boundary treatment shows stable convergence regardless of the boundary shape. We will demonstrate the method's stability and accuracy with various numerical results that we compare with exact solutions and numerical results having second-order accuracy.

2. Lattice Boltzmann method (LBM) with single relaxation time

A popular kinetic model of LBM is the BGK model with a single relaxation time [1,3,5] as follows:

$$\frac{\partial f}{\partial t}+\vec{\xi}\cdot\nabla f=-\frac{1}{\lambda}\left(f-f^{(0)}\right), \tag{1}$$

where $f^{(0)}$ is the equilibrium distribution function (Maxwell–Boltzmann distribution function), $f(\vec{x},\vec{\xi},t)$ is the particle velocity distribution function, $\vec{x}$ is the spatial position vector, $\vec{\xi}$ is the particle velocity vector, t is the time, and λ is the relaxation time. To solve the particle velocity distribution numerically, Eq. (1) is discretized by the discrete-velocity set $\vec{e}_\alpha$ [14,15].

$$\frac{\partial f_\alpha}{\partial t}+\vec{\xi}_\alpha\cdot\nabla f_\alpha=-\frac{1}{\lambda}\left(f_\alpha-f_\alpha^{(eq)}\right), \tag{2}$$

where $f_\alpha(\vec{x},t)$ is the distribution function with the α^{th} discrete velocity $\vec{e}_\alpha$ and $f_\alpha^{(eq)}$ is the corresponding equilibrium distribution function in the discrete velocity space.

Eq. (2) is discretized with the time step δt and space step $\delta x=\vec{e}_\alpha\delta t$ into

$$\begin{aligned} f_\alpha(\vec{x}_i+\vec{e}_\alpha\delta t,t+\delta t)-f_\alpha(\vec{x}_i,t)&=-\frac{1}{\tau}\left[f_\alpha(\vec{x}_i,t)-f_\alpha^{(eq)}(\vec{x}_i,t)\right],\\ f_\alpha(\vec{x}_i+\vec{e}_\alpha\delta t,t+\delta t)&=f_\alpha(\vec{x}_i,t)+\Omega_\alpha(\vec{x}_i,t), \end{aligned} \tag{3}$$

where $\tau\left(=\frac{\lambda}{\delta t}\right)$ is the non-dimensionalized relaxation time, x_i is the point in the discretized physical space, and $\Omega_\alpha\left(=-\frac{1}{\tau}\left(f_\alpha-f_\alpha^{(eq)}\right)\right)$ is the collision term. Eq. (3) is usually solved with the following two steps.

(1) Collision step

$$\tilde{f}_\alpha(\vec{x}_i,t+\delta t)=f_\alpha(\vec{x}_i,t)-\frac{1}{\tau}\left[f_\alpha(\vec{x}_i,t)-f_\alpha^{(eq)}(\vec{x}_i,t)\right]. \tag{4}$$

(2) Streaming step

$$f_\alpha(\vec{x}_i+\vec{e}_\alpha\delta t,t+\delta t)=\tilde{f}_\alpha(\vec{x}_i,t+\delta t), \tag{5}$$

where $\tilde{f}_\alpha$ is the post-collision state of the distribution function. These two steps are localized and simple to implement. We used a nine-velocity set model for two-dimensional problems (D2Q9) and a 19-velocity set model for three-dimensional problems (D3Q19) [5]. The equilibrium distributions for D2Q9 and D3Q19 have similar forms.

$$f_\alpha^{(eq)}=\rho w_\alpha\left[1+\frac{3}{c^2}(\vec{e}_\alpha\cdot\vec{u})+\frac{9}{2c^4}(\vec{e}_\alpha\cdot\vec{u})^2-\frac{3}{2c^2}(\vec{u}\cdot\vec{u})\right], \tag{6}$$

where $\vec{e}_\alpha$ is a discrete-velocity set. The term $\vec{e}_\alpha$ is nine-velocity set in D2Q9: $\vec{e}_\alpha=0$ for $\alpha=0$, $\vec{e}_\alpha=c(\pm1,0)$ for $\alpha=1,2$, $\vec{e}_\alpha=c(0,\pm1)$ for $\alpha=3,4$, $\vec{e}_\alpha=c(\pm1,\pm1)$ for $\alpha=5,6,7,8$, where $c\left(=\frac{\delta x}{\delta t}\right)$ is the lattice speed, δx is the lattice constant, δt is the time step, and w_α is a weighting factor as follows: $w_\alpha=4/9$ for $\alpha=0$, $w_\alpha=1/9$ for $\alpha=1,2,3,4$, $w_\alpha=1/36$ for $\alpha=5,6,7,8$. The term $\vec{e}_\alpha$ is 19-velocity set in D3Q19: $\vec{e}_\alpha=0$ for $\alpha=0,\vec{e}_\alpha=c(\pm1,0,0)$ for $\alpha=1,2,\vec{e}_\alpha=c(0,\pm1,0)$ for $\alpha=3,4,\vec{e}_\alpha=c(0,0,\pm1)$ for $\alpha=5,6$, $\vec{e}_\alpha=c(\pm1,\pm1,0)$ for $\alpha=7,8,9,10$, $\vec{e}_\alpha=c(\pm1,0,\pm1)$ for $\alpha=11,12,13,14$, $\vec{e}_\alpha=c(0,\pm1,\pm1)$ for $\alpha=15,16,17,18$, and w_α is a weighting factor as follows: $w_\alpha=1/3$ for $\alpha=0$, $w_\alpha=1/18$ for $\alpha=1,2,\ldots,6$, $w_\alpha=1/36$ for $\alpha=7,8,\ldots,18$. The formations of a lattice for the D2Q9 and D3D9 are depicted in Fig. 1.

In addition, there are diverse forms of the equilibrium distribution depending on the discrete-velocity sets. The physical properties, such as density and momentum, are determined from the distribution function used as a primary variable in the LBM. The density is evaluated by $\rho=\sum_{\alpha=0}^{r}f_\alpha=\sum_{\alpha=0}^{r}f_\alpha^{(eq)}$, the specific momentum $\rho\vec{u}=\sum_{\alpha=1}^{r}\vec{e}_\alpha f_\alpha=\sum_{\alpha=1}^{r}\vec{e}_\alpha f_\alpha^{(eq)}$, the pressure $p=\rho c_s^2=\rho\frac{c^2}{3}$, the viscosity $\nu=\left(\tau-\frac{1}{2}\right)c_s^2\delta t$, and the speed of sound $c_s=\frac{c}{\sqrt{3}}$.

3. Boundary condition for a curved wall

The half-away bounce-back boundary condition is easy to implement and commonly used. This boundary condition can be interpreted based on the finite cell concept, as shown in Fig. 2.

All cells have the same size, and the lattice points are located at the center of each cell. The size of the cell near a wall is the same as one in the inner fluid region. With this constraint on cell size, the half-away bounce-back boundary condition can maintain second-order accuracy [8], but the wall geometries cannot be accurately reflected with a uniform cell size. In order to calculate complex wall geometries, we need to increase the number of lattice points and put significant limits on the geometries.

A simple geometry with uneven cell size at the boundary is depicted in Fig. 3. It is commonly accepted that non-uniform cell sizes at the boundary provide first-order accuracy when using the typical half-away bounce-back treatment.